Kalman filtration, filter algorithm, accelerometric sensor

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THE USE OF KALMAN FILTRATION TO ESTIMATE CHANGES OF TRUNK INCLINATION ANGLE DURING WEIGHTLIFTING

The paper presents a concept of a sensor of weightlifter's trunk inclination angle with the use of Kalman filter algorithms to estimate the trunk inclination angle. The paper presents changes of trunk inclination angle obtained using the algorithm presented. The application of an accelerometric and gyroscopic sensor in the measuring system combined with the algorithms presented in the paper enables precise representation of angle changes during the exercise.

1. INTRODUCTION

Considerable forces act on the vertebral column during the weightlifting, which magnitude and influence on the vertebrae structure depend on a number of factors. The force of gravity acting on the top part of weightlifter's body and the weight held by him may be considered as a component compressing the vertebrae and intervertebral discs and a component, which tends to shift forward a higher vertebra against a lower vertebra. This force causes a bending load on the vertebral arch, which rotates inferior articular processes backwards.

The interaction line of gravity forces acting on the centre of mass of the upper part of the body and the barbell held goes from the front in relation to the lumbar-sacral section, forming this way a bending torque, which must be opposed by muscles straightening the vertebral column and the hip joints. The tone of sacrodorsal muscles causes a force acting upwards on the spinous processes and the vertebral arch. This force is opposite to the gravity force component acting on the arch at its base.

The total extension force may be estimated based on the anthropometric data related to the mass and position of the vertebral column and trunk. The reaction to the extension force releases a force pressing on the vertebral bodies and intervertebral discs at simultaneous force resulting from the gravity force [2]. For example, for an individual with the mass of the upper half of the body equal to 60 kg and the weight equal to 80 kg in an inclined position the total compressing force is around 8500 [N]. This is a force, which may overload a vertebral body and an intervertebral disc. The mechanism of increase in the pressure of abdominal press is a factor weakening the intervertebral pressure. During weightlifting an increase in the pressure in the abdominal cavity depends on the speed of lifting and on the size of the weight lifted. In athletes participating in competitive weightlifting the pressures of the abdominal press amount to ca. 350 mm Hg. A pressure of this order may reduce the compressing forces to around 10,000 [N] provided that the vertical weightlifting starts from a static position. However, improving of a sport result requires that an athlete accumulates the kinetic energy before starting the lifting.

The pressure increase in the abdominal cavity peaks simultaneously with the peak of weight acceleration and is proportional to the total load acting on the vertebral column. The pressure peak is momentary and is most effective in reducing the pressure on the vertebral column at the moment, when the lumbar section of the vertebral column is inclined. The peak of vertebral column load and the peak of the pressure in the abdominal cavity coincide at the beginning of the lifting phase, before the process of straightening the lumbar section of the vertebral column starts. The lowest forces occur at the moment, when the centre of barbell mass is held close to the body and the vertebral column is inclined. The influence of load on the vertebral column strength is directly proportional to the load's duration.

The angle of trunk inclination has a substantial influence on the vertebral column load. The measurement of this parameter combined with the knowledge of loads is vital to determine the influence of exercising on the condition of a kinematic chain, which is the vertebral column.

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2. MEASURING SYSTEM DESIGN

The measuring system has been equipped with two sensors. The first of them enables the measurement of acceleration in two axes, perpendicular to each other. At the moment of assuming an erect position by the object the first axis is perpendicular and the second parallel to the base [1]. The second sensor enables measurement of the angular velocity during a bend and extension at the height of S5-L1 vertebrae. The task of an accelerometric sensor consists of determination of reference points of the measurement, when the object is still and the sensor is affected by the vector of gravitational acceleration only. The measuring system determines the reference angle of sensor position acc. to the following relationship:

$$\alpha = \arccos(ax) \tag{1}$$

$$\text{or } \alpha = \arcsin(ay) \tag{2}$$

where:

ax – the value of acceleration registered by accelerometer x
ay – the value of acceleration registered by accelerometer y

To determine the moment, at which it is possible to start determining the reference angle it is necessary to find such a point in the measuring vector, which fulfils the following conditions:

$$C \subseteq D$$

$$\begin{aligned}
&1. \quad (\forall n \in C)\,|\max(\omega_n) - \min(\omega_n)| < \tau \\
&2. \quad (\forall n \in C)\,|\max(ax_n) - \min(ax_n)| < \gamma_x \\
&3. \quad (\forall n \in C)\,|\max(ay_n) - \min(ay_n)| < \gamma_y \\
&4. \quad (\forall n \in C)\sqrt{ax_n^2 + ay_n^2} - 1 < \delta
\end{aligned} \tag{3}$$

where:

D - a set of all indices of measuring points;
C - a set of n subsequent indices;
ω - the angular velocity;
ax_n, ay_n - the linear acceleration in point n;
τ - the maximum permissible drift of the gyroscope;
γ_x, γ_y - the maximum permissible deviation of the acceleration for a static state;
δ - the maximum error of the measuring system deviation in plane z.
n - number of sample in measurement.

The size of set C affects the number of reference points fixed in the results matrix, for which it is possible to state that the object is in a static state. The increase in the set C size may result in a situation, in which the reference points will not be detected. The decrease in the set will result in the occurrence of many “false” reference points. The size of the set C was experimentally determined as 20 indices, which corresponds to a frame with a measuring time of 0.2 s. Fig. 1 presents the position of reference points for various sizes of the set C. It can be noticed that for a frame of size 20 three reference points have been detected for the whole measurement, which correspond to situations, in which the object is erect, inclined, and erect again.

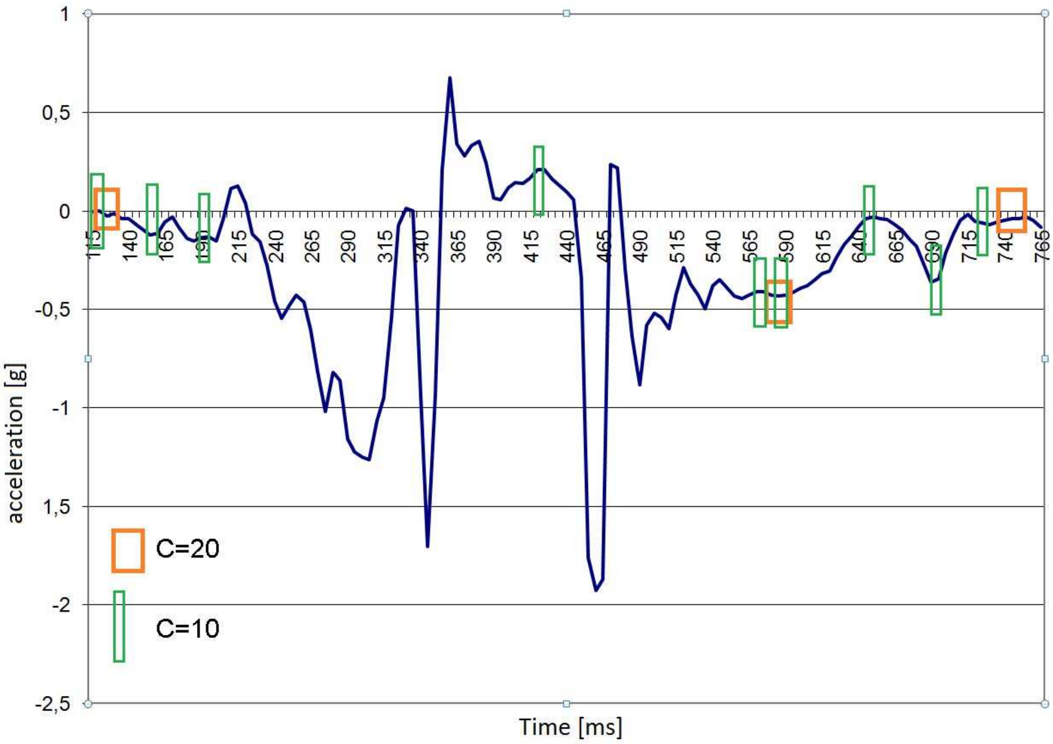

Fig.1. Positions of reference points.

3. KALMAN FILTER APPLICATION TO DETERMINE THE INCLINATION ANGLE

3.1. PROCESS AND MEASUREMENT MODEL

The Kalman filtration enables determining variables, inaccessible by measurements, based on the current values of magnitudes inaccessible by measurements and on the knowledge of a mathematical model interconnecting individual measurements. The Kalman filters feature the following characteristics[3]:

- A Kalman filter is an optimal estimator, because under specific assumptions it may fulfil certain criterion, e.g. the minimisation of mean-square error of parameters estimated;
- It is possible to use all accessible measurements irrespective of the accuracy of their performance;
- A Kalman filter does not store all the past data and therefore there is no need to recalculate all measurements in each step. The information calculated in the previous step is used for calculations;
- Knowing the system input and output it is possible to obtain the information, which is inaccessible (immeasurable) based on the information available from e.g. sensors;
- The Kalman filtration method is an optimal estimator of the state, because it enables obtaining as optimal as possible value based on many measurements originating from a noisy environment.

The following mathematical model may be used to describe the measured and the measuring systems:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \tag{4}$$

$$z_k = Hx_k + v_k \tag{5}$$

Equation (4) presents a process model, in which the first part is of a deterministic and the second of a random nature. The connection with the previous state occurs via matrix *A*, matrix *B* represents the control, and w_k is the process noise. Equation (5) represents a measurement model, in which *H* is a matrix connecting the measurement with the state (filter output), v_k similarly to w_k represents the disturbance in the form of the Gauss noise.

3.2. DISCRETE ALGORITHM FOR KALMAN FILTRATION

A Kalman filter is a two-phase recursive algorithm, in which phase one is named a prediction and phase two a correction. The prediction phase uses equations of time updating, during which the estimated value of state $\tilde{x}_{\bar{k}}$ and its covariance are determined using the state from the previous step. The measurement update is performed in the correction phase.

$$\begin{gathered} \tilde{x}_{\bar{k}} = A\tilde{x}_{k-1} + B\tilde{u}_{k-1} \\ P_{\bar{k}} = AP_{k-1} \cdot A^T + Q \end{gathered} \tag{6}$$

Equation (6) presents the prediction phase, where $\tilde{x}_{\bar{k}}$ and $P_{\bar{k}}$ are forecast a priori values of state and covariance, $\tilde{x}_{\bar{k}-1}$ and P_{k-1} are optimal estimated a posteriori values performed in the previous step. The influence of the correction phase on the state estimated is the Kalman gain [3].

$$K_k = P_k \cdot H^T (HP_k \cdot H^T + R)^{-1} \tag{7}$$

Equation (8) presents the value of optimal forecast correction during time k based on all hitherto measurements.

$$\tilde{x}_{\bar{k}} = \tilde{x}_{\bar{k}} + K_k (z_k - H \cdot \tilde{x}_{\bar{k}}) \tag{8}$$

where:

z_k - the measured value, $(z_k - H \cdot \tilde{x}_{\bar{k}})$ measurement innovation.

The covariance matrix may be corrected as follows:

$$P_k = (I - K_k H) P_k \tag{9}$$

where:

I - a unit matrix.

3.3. MEASUREMENT OF STATIC VALUE OF THE INCLINATION ANGLE

The measuring system equations may assume the form presented in equations (4) and (5). In the case of measurement, in which there is a static angle, i.e. which value does not change in time, *A*=1 may be assumed. It is possible to estimate that in each next step the angle will have the same value. As there is no input controlling the system it is possible to assume *u*=0 due to the fact that the measurement and state have dimension equal to one, matrix H=1. Equations (4) and (5) will be simplified and they may be presented in the following form:

$$\tilde{x}_{\bar{k}} = \tilde{x}_{k-1} \tag{10}$$

$$P_{\bar{k}} = P_{k-1} + Q$$

After simplification equation (7) will take the following form:

$$K_k = P_{\bar{k}}(P_{\bar{k}} + R)^{-1} \tag{11}$$

For a constant angle variance Q should basically be equal zero, however, as it may be found in paper [3] the assuming of even very small value of the parameter (of the order of 10e-5) can adjust the filter in a better way.

Fig. 2 presents the measurement of static angle value for variance R=0.001 and Q=10e-5.

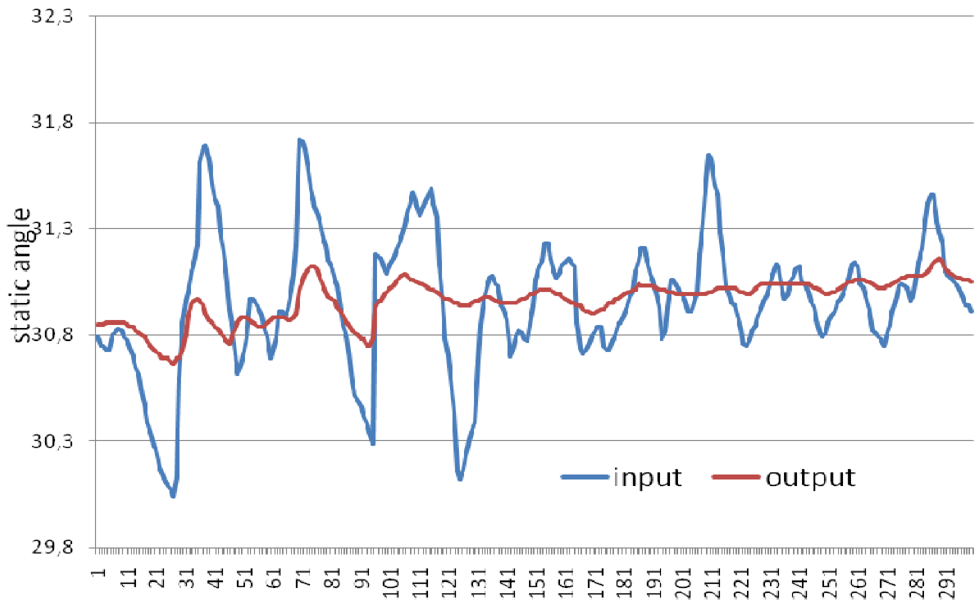

Fig. 2. Measurement of static angle.

3.4. MEASUREMENT OF DYNAMIC VALUE OF THE INCLINATION ANGLE

The use of Kalman filtration to measure a dynamic value of the inclination angle may be similar to the case of static measurement. In this case the value of A=1 and H=1 may be assumed and parameter Q must take a value a few orders of magnitude higher than in the case of a static measurement.

As Fig. 3 shows, certain phase shift appears in relation to the actual angle measurement. This is an unfavourable phenomenon, which may be eliminated by the estimation of angle and of angular velocity.

$$\theta_k = \theta_{k-1} + (\omega_{k-1} + w_{k-1})dt$$

$$\omega_k = \omega_k + v_k \tag{12}$$

where:

θ - the inclination angle value, ω - the angular velocity.

The system matrix has the form:

$$A = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix} \tag{13}$$

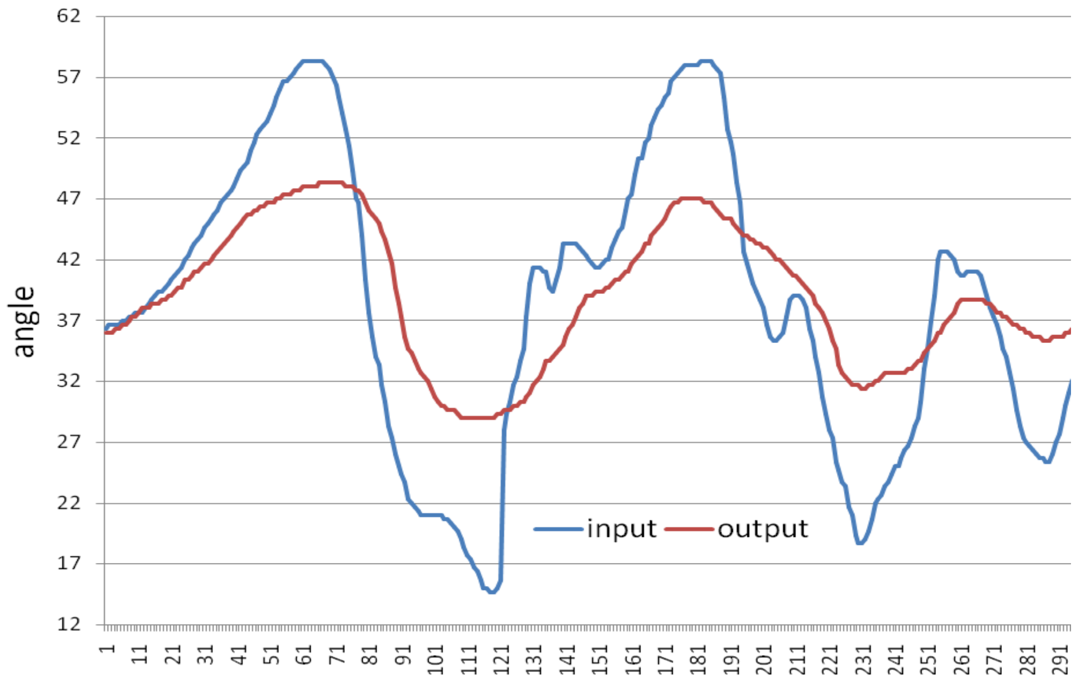

Fig. 3. Phase shift appears in relation to actual angle.

The vector of state

$$x = \begin{bmatrix} \theta \\ \omega \end{bmatrix} \tag{14}$$

The filter output:

$$H = [1 \quad 0] \tag{15}$$

The covariance matrix Q:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} q \tag{16}$$

where:

Q - the process variance.

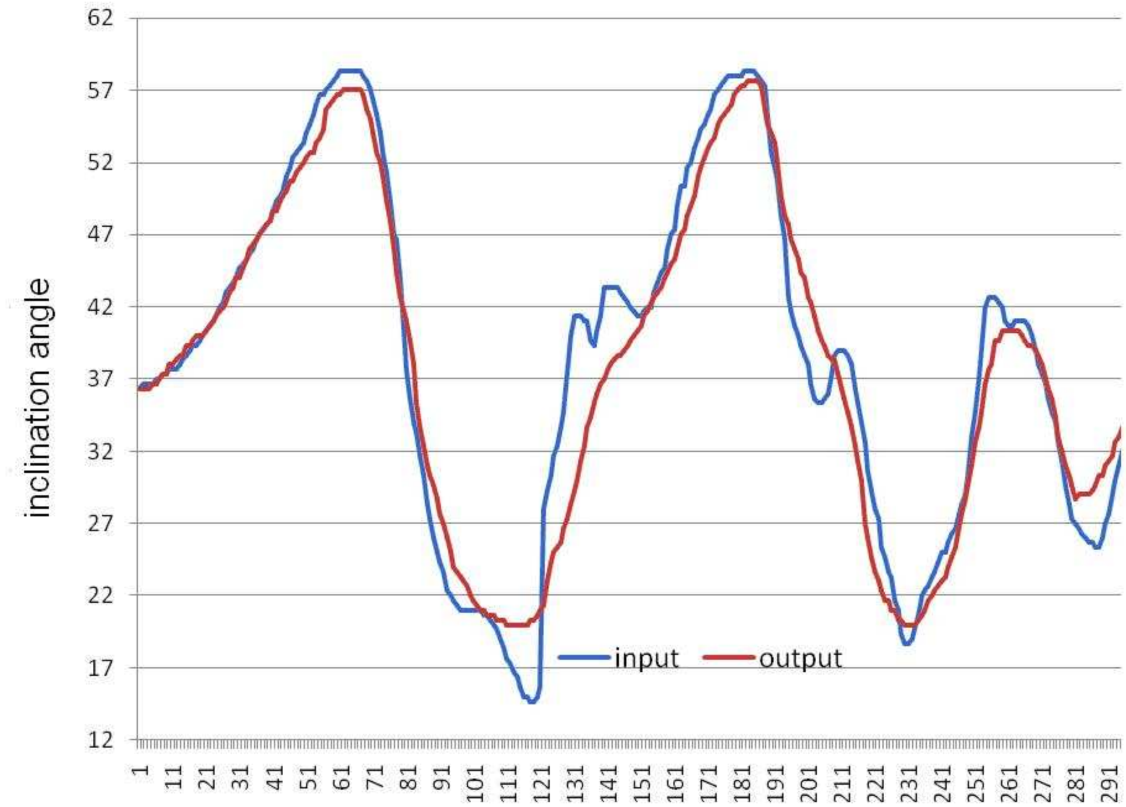

Fig. 4. The inclination angle estimation for *R*=200 and *Q*=0.01.

Fig. 4 presents the inclination angle estimation for *R*=200 and *Q*=0.01.Despite the application simplicity the use of a gyroscope to measure the angular velocity generates some problems related to an unfavourable effect such as drift, i.e. so-called zero shift in time. The drift is not a random but a systematic error. Using equation (4) the angular velocity ω read from the gyroscope may be presented as control *u*.

$$\theta_k = A\theta_{k-1} + B\omega \tag{17}$$

Matrix *A* will assume the value as in equation (13). Matrix *B* will assume the following value

$$B = \begin{bmatrix} dt \\ 0 \end{bmatrix} \tag{18}$$

Vector of state x:

$$x = \begin{bmatrix} \theta \\ bias \end{bmatrix} \tag{19}$$

Covariance matrix Q will assume the form from equation (2). Measurement z_k originates from tangent points of the angle determined by the algorithm searching for values of angles measured by the accelerometer.

4. CONCLUSIONS

The algorithms of an enhanced Kalman filter presented in the paper provide an interesting solution for the estimation of the trunk inclination angle during exercises related to weightlifting by athletes. The Kalman filtration cancels standard problems related to the measurement with the use of gyroscopes, such as a voltage drift and the measuring noise. The application of a mobile wireless measuring system improves the comfort of exercising by weightlifters and the measurement results obtained combined with additional measuring systems such as an extensometric platform enable eliminating improper characteristics of athlete's behaviour, which can result in serious injuries.

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