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PERFORMANCE EVALUATION OF BALDWIN'S FUZZY REASONING FOR LARGE KNOWLEDGE BASES

The paper compares performance of Baldwin's fuzzy reasoning based on a fuzzy truth value with the fastest available solutions. The analysis is important in order to locate areas where improvement of the first is the most significant. Potential fast approach based on the fuzzy truth value would be very interesting for many users applying fuzzy systems to solve problems involved with complex knowledge bases. Particularly, all research considering an analysis of genes employing DNA microarrays. Such methods very often generate rules with thousands of atomic premises.

The most valuable advantage of Baldwin's reasoning is preserving a fuzzy relation between a fact and a premise in the inference process, where other solutions, especially those commonly used, usually reduce it to only one value. Obtaining the method which, from computation time point of view, is comparable with common approaches but offers more advanced process of fuzzy reasoning, would be a significant achievement.

The goal of this analysis is to prepare the future research considering development of Baldwin's method, which computational complexity is comparable to simple, fast and widely used solutions like systems based on the approach of Mamdani and Assilan or Larsen.

1. INTRODUCTION

Among a great number of different applications, fuzzy systems are widely employed in miscellaneous areas of medicine and biology. The whole environment of medical problems is extremely uncertain. That is why the soft computing methods are best suited to such domain.

Some complex problems involve a large amount of data to be analyzed and processed. In this case the knowledge base of a fuzzy system contains many rules with compound premise (even thousands of atomic premises). Good examples in such area are genes-related problems, which are extensively studied by many scientists using different fuzzy approaches [4-6, 8-11, 16-17, 19-27]. Many research problems in this area are involved with the analysis of DNA microarrays used in simultaneous measurement of DNA expression levels or to genotype multiple regions of a genome. The knowledge bases of fuzzy systems in this case often contain complex rules with thousands of atomic premises.

This paper contains the analysis of computational complexity comparing widely used and fast reasoning approaches with the forgotten method of Baldwin. Subsequent sections describe mentioned solutions, contain analysis of numerical implementations with conclusion at the end, where further research directions are proposed.

2. FUZZY INFERENCE AND THE FUZZY TRUTH VALUE

Fuzzy reasoning based on the fuzzy truth value was introduced by Baldwin in 1979 [1], which was a few years after the first approach, compositional rule of inference, described by Zadeh in 1973 and 1975 [28-30]. The main idea differentiating these two solutions is the reasoning domain. The classic compositional rule of inference works directly on the truth functions of a fact and a premise. The idea for modus ponendo ponens can be expressed by the following equation [7]:

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$$\mu_{B'}(y) = \sup_{x \in X} \left(\mu_{A'}(x) *_T \ I\left(\mu_A(x), \mu_B(y)\right)\right), \tag{1}$$

where $\mu_{A'}(x)$, $\mu_A(x)$, $\mu_{B'}(y)$, $\mu_B(y)$ are membership functions, $*_T$ represents any triangular norm used for intersection between a fuzzy fact A' and a fuzzy implication I, which is obtained from a fuzzy premise A and a fuzzy conclusion B. The fuzzy sets are described in universes of discourse X, for a premise and a fact, and Y for the conclusion B and the result of the inference B'. Computational complexity of the approach (1) for multi-compound premises becomes problematic because of multi-dimensional analysis, which is shown below for only two premises [7]:

$$\mu_{B'}(y) = \sup_{x_1 \in X_1, x_2 \in X_2} \left(\mu_{A_1'}(x_1) *_T \ \mu_{A_2'}(x_2) *_T \ I\left(\mu_{A_1}(x_1) *_T \ \mu_{A_2}(x_2), \mu_B(y)\right)\right). \tag{2}$$

A very common simplified approach was presented for the first time in 1975 by Mamdani and Assilan [18]. The idea used minimum operation as a triangular norm and values of facts (system input) were fuzzyfied with singleton, which is described by the following equation for two premises and facts [7]:

$$\mu_{B'}(y) = \min\left[\min\left(\mu_{A_1}(x_{in1}), \mu_{A_2}(x_{in2})\right), \mu_B(y)\right], \tag{3}$$

where x_{in1} and x_{in2} represent input values of the fuzzy system (facts like 50 km/h or -4°C). Larsen [15] proposed a very similar approach that used product instead of minimum operation.
It can be noticed that for calculating one value of $\mu_{B'}(y)$ membership function the analysis of X_1 and X_2 domains is not longer performed. It is very important to emphasize, that (3) is not equivalent to (2) considering all available implications, triangular norms and different methods of input fuzzyfication and types of membership functions. In general the results obtained for this two equations are different, especially for fuzzyfied input and within the ranges of y variable, where $0<\mu_B(y)<1$.

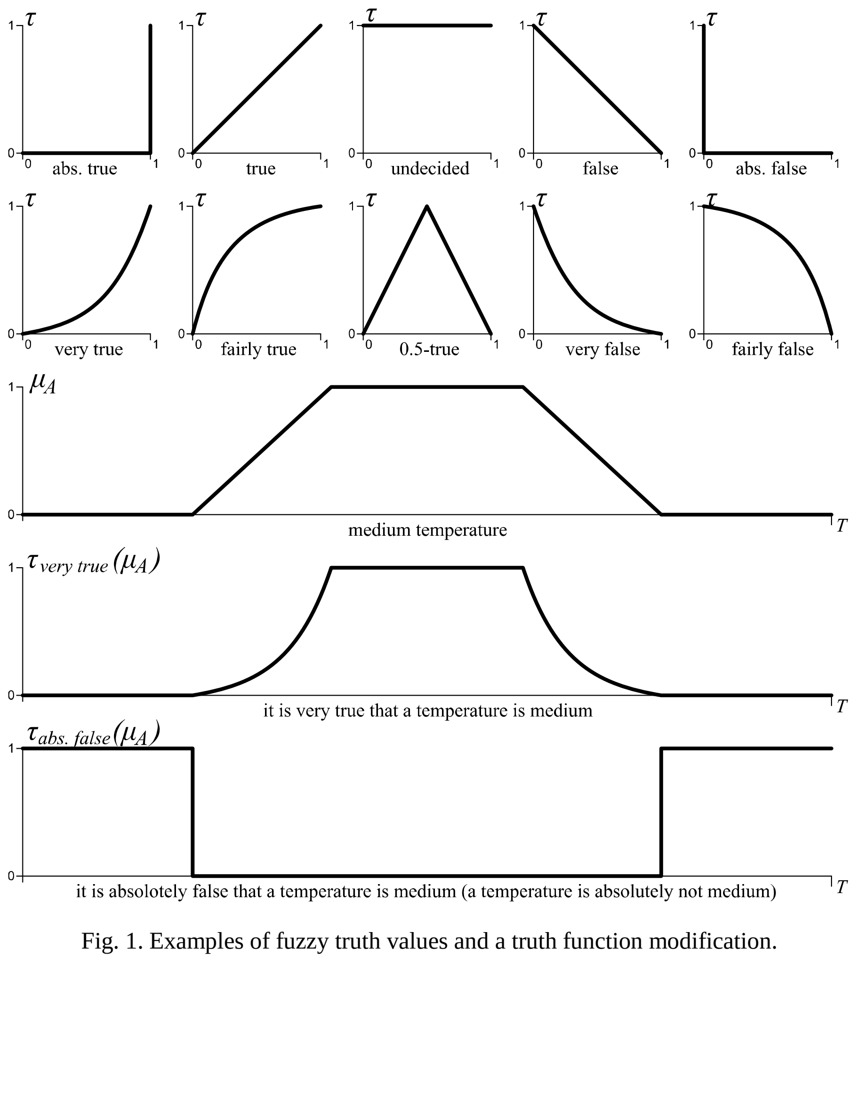

Fig. 1. Examples of fuzzy truth values and a truth function modification.

The fuzzy reasoning presented by Baldwin [1] moves the inference process into the space of the fuzzy truth values [3,1]. Initially, corresponding facts and premises are combined to obtain, so called, truth function of a premise τ_p (membership function of a fuzzy truth value) [1]:

$$\underset{\eta\in[0,1]}{\forall} \quad \tau_p(\eta) = \sup_{\substack{\eta=\mu_A(x)\\ x\in X}} \left[\mu_{A'}(x)\right]. \tag{4}$$

Calculated τp represents compatibility of a particular fact with corresponding premise. Depending on the level of compatibility and types of μA'(x), μA(x) membership functions, τp can take infinite number of forms. Generally, some basic examples were named like absolutely true (τabs.true), true (τtrue), undecided (τund.), false (τfalse), very true (τvery true), fairly true (τfair.true), fairly false (τfair.false) and very false (τvery false) [1].

The truth function can be directly used to modify any linguistic expression [3]. For example:

$$\underset{x\in X}{\forall} \quad \mu_{A\ very\ true}(x) = \tau_{very\ true}(\mu_A(x)). \tag{5}$$

Therefore, assuming a linguistic expression *A* like "medium temperature", the new expression $A_{very\ true}$ is obtained: "it is very true that temperature is medium". The truth function modification [3] can completely change the meaning of an expression like "it is absolutely false that temperature is medium". Fig. 1 presents mentioned truth functions and how they can be used to modify fuzzy expressions.

Reasoning phase in the method of Baldwin is based only on the fuzzy truth space. Corresponding premises and facts are transformed by (4) into truth functions. In case of fuzzy rules containing compound premise (i.e. "air <u>temperature is medium</u> and air <u>humidity is high</u>") all atomic premises can be sequentially joined into one collective truth function τ_p by the following equation [1]:

$$\underset{z\in[0,1]}{\forall} \quad \tau_p(z) = \sup_{\substack{x\, *_N\, y=z\\ x,y\in[0,1]}} \left[\tau_{p_1}(x) *_T \tau_{p_2}(y)\right], \tag{6}$$

where $*_N$ indicates either triangular norm (T-norm) or triangular conorm (S-norm), depending on a type of conjunction (AND or OR respectively).

The truth function of a premise (τ_p) is used to obtain the truth function of a conclusion (τ_q), according to the following equation [1]:

$$\underset{\phi\in[0,1]}{\forall} \quad \tau_q(\phi) = \sup_{\eta\in[0,1]} \left[\tau_p(\eta) *_T I(\eta,\phi)\right], \tag{7}$$

where *I* represents any fuzzy implication or triangular norm in case of conjunctive approach.

Generally speaking, τ_q represents *"truth"* only when premise is true. Otherwise it takes more or less the form of *"undecided"* truth function ($\tau_{und.}$). This leads to the final phase of reasoning, where result *B'* is calculated basing on conclusion *B*. With τ_q function the conclusion can be obtained from *B* using the truth function modification, like it was described earlier [3,1]:

$$\underset{y\in Y}{\forall} \quad \mu_{B'}(y) = \tau_q(\mu_B(y)) . \tag{8}$$

Transforming domains of all different premises into one unified truth space can be perceived as an advantage from computational point of view (comparing to the compositional rule of inference without modifications). The problem of compound premise is no longer complex because in this case the reasoning process is characterized by linear computational complexity according to the number of atomic premises in a rule, which is precisely analyzed in subsequent sections.

It is important to emphasize, that this solution preserves a full fuzzy relation between facts and premises (in the form of truth functions) through the whole reasoning process. In simplified approaches this relation is mapped to only one value in [0,1] range. Nevertheless, the computational complexity of the solution remains much higher in comparison to the approaches like (3). Therefore, an analysis finding area of the best possible improvement is very important for future development of the solution, which on the one hand is much more efficient but on the other hand preserves all the advantages mentioned earlier.

3. COMPUTATIONAL COMPLEXITY ASSESSMENT

Precise analysis of computational complexity in a general case is impossible, because the problem is closely related to a particular algorithm performing the designed task. Therefore, some assumptions have to be made to confront and assess described methods.

Fuzzy sets' representation for calculation purposes can be various. Contemporary solutions use different approaches to describe and store membership functions. The most common are α-cuts, piece-wise linear functions, discrete points and mathematical expressions (i.e. exact description of Gaussian function). Mathematical expressions are usually the fastest at the first phases of inference process (obtaining a membership value), but can be problematic to precisely describe obtained conclusion, especially after aggregation of the results. The other approaches are flexible at any phase of calculation. However, they are usually involved with higher size of stored data, depending on the accuracy of represented membership functions.

For the rough assessment of computational complexity it was assumed, that a membership function is described by N elements, either discrete points, nodes of piece-wise linear function or number of analyzed α-cuts. Such assumptions let to evaluate possible algorithms in terms of number of needed operations.

3.1. SIMPLE AND FAST APPROACHES

Considering the solution of Mamdani and Assilan based on equation (3), the computational complexity of the approach can be described by the following expression:

$$K \log_2 N + (K-1) + N, \tag{9}$$

where

$\log_2 N$: represents calculations needed to obtain a value of membership function described by ordered N elements (for either premise or conclusion),

K : represents a number of atomic premises in a compound premise

Analyzing the expression (9) it can be noticed that the first part is responsible for calculating K values for atomic premises, the second for calculating compound membership level (junctions of K values from the first part) and the third for obtaining a fuzzy result (based on the fuzzy set of conclusion described by N elements). Therefore, computational complexity depending on the N parameter can be expressed in big O notation by $O(N)$ for small K values and $O(log_2 N)$ for high K values and $O(K)$ for the analysis depending on the K parameter.

In contrary to complexity of (2), presented levels clearly indicate linear relation with number of atomic premises in a rule.

To verify the validity of the expression (9) according to N parameter, two groups of tests were performed. The first group concerned a greater influence of the logarithmic part for high values of K. The second group verified a linear relationship in case of low K values.

Fig. 2 presents results obtained for $K=10^4$ and Fig. 3 for $K=10$. Membership functions of atomic premises and conclusions used in all tests were Gaussian. Each result shown on Fig. 2 represent computation time for 100 rules and results from Fig. 3 represent time obtained for 5000 rules.

It can be noticed, that the Fig. 2 clearly demonstrates the influence of logarithmic part and the Fig. 3 confirms the linear dependence.

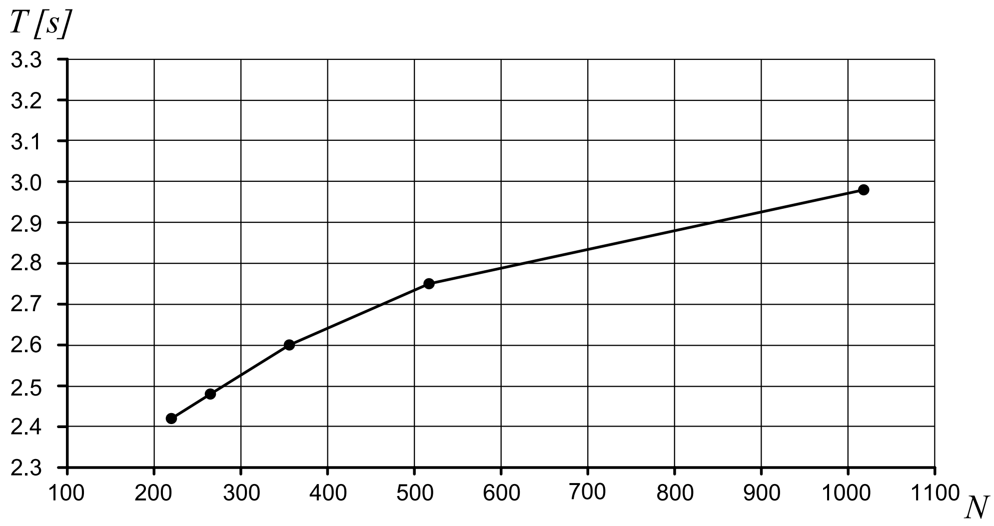

Fig. 2. Influence of *N* parameter on computation time for fast systems with K=10[3].

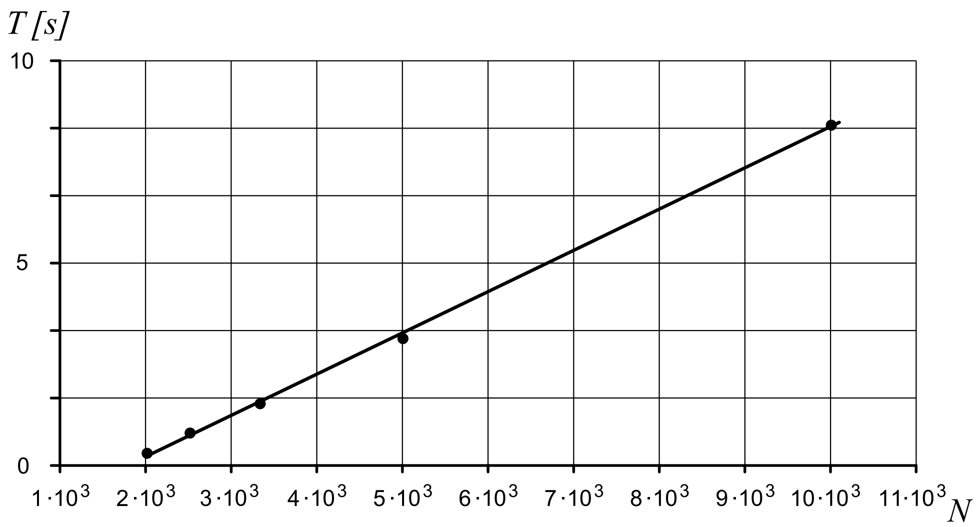

Fig. 3. Influence of *N* parameter on computation time for fast systems with *K*=10.

The last group of tests were performed to verify a linear complexity of the problem according to *K* parameter. In this case the *N* parameter was fixed and equal 2023. The results are presented in Fig. 4 and contain average time of computations for one rule.

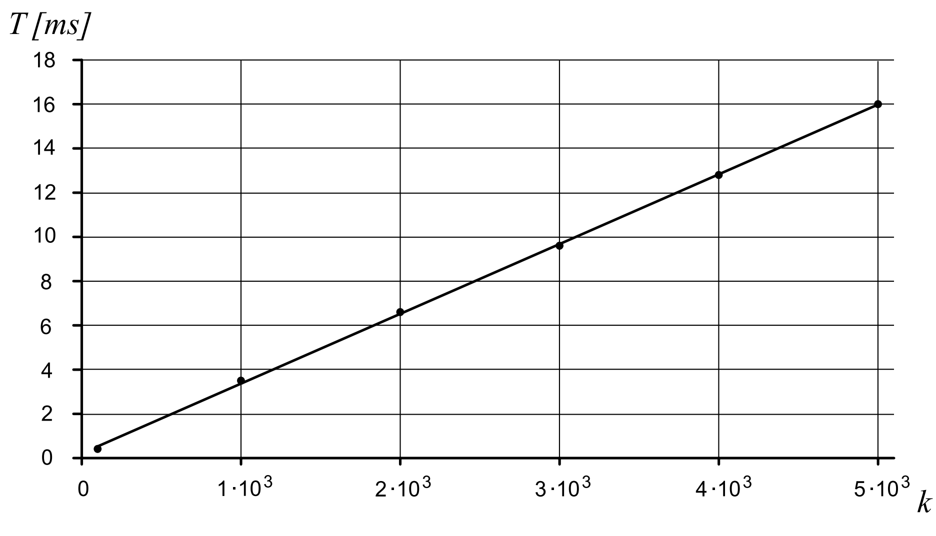

Fig. 4. Influence of *K* parameter (number of atomic premises in a rule) on computation time for fast systems.

3.2. THE APPROACH OF BALDWIN

Computational complexity of Baldwin's reasoning strongly depends on the algorithms obtaining the described truth functions. Baldwin proposed a simple discrete solution [2] and since 1980 it is very hard to find other implementations, because the approach seems to be forgotten. The author of this article proposed within the Fuzzlib library [12-14] the solution using piece-wise linear representation of membership function. The library contains algorithms constructing truth functions according to chosen precision parameter *minDY*. This allows the system user to control accuracy of representation. For fixed *minDY* the number of elements describing generated truth functions encloses within a small range and it does not depend on the number of elements describing membership functions of facts and premises. For example, when $minDY=10^{-2}$ the number of elements describing generated truth function is approximately 19. In this case computational complexity of obtaining a premise truth function takes the following form

$$C_{\min DY} \log_2 N = O(\log_2 N), \tag{10}$$

where C_{minDY} represents the number of elements describing a truth function depending on *minDY* parameter. For fixed *minDY* it can be considered as constant and it can be omitted in the big O notation.

The next phase of Baldwin's reasoning involves junctions of truth functions obtained for atomic premises. The result represents the truth function of a compound premise, which is used in subsequent phases of the inference.

The Fuzzlib library proposes the approach where in order to calculate one point of an output truth function the description of two combined truth functions is iteratively analyzed. Therefore, the solution can be described by the following expression:

$$2C_{\min DY} C_{\min DY} = 2{C_{\min DY}}^2, \tag{11}$$

where $2C_{minDY}$ represents analysis of two input truth functions, which is performed C_{minDY} times to calculate elements of an output truth function.

Computational complexity of the next phase in Baldwin's reasoning, which is obtaining a truth function of conclusion described by (7), is at the same level as computing a compound truth function (11). It also considers calculations of C_{minDY} elements, where analysis of a premise truth function is needed for each element. Therefore, it can be expressed by the following expression:

$$C_{\min DY} C_{\min DY} = {C_{\min DY}}^2. \tag{12}$$

The last phase, described by (8), is involved with obtaining the fuzzy result *B'* basing on the fuzzy conclusion *B*, which is the truth function modification. This stage considers obtaining a truth function value ($\log_2(C_{minDY})$) executed *N* times, which directly leads to the expression:

$$N \log_2 (C_{\min DY}). \tag{13}$$

Therefore, considering all elements of Baldwin's inference for one fuzzy rule, the computational complexity can be described by the following expression:

$$KC_{\min DY} \log_2 N + (K-1)2{C_{\min DY}}^2 + {C_{\min DY}}^2 + N \log_2 (C_{\min DY}), \tag{14}$$

which respectively includes: obtaining *k* truth functions of atomic premises, (*k-1*) junctions of premise truth functions, obtaining a truth function of conclusion and the final fuzzy result.

Similarly to (9), it can be noticed that (14) is characterized by linear dependence according to *K* and *N* parameters. These assumptions were verified by numerical tests, the results of which are presented at Fig. 5 and Fig. 6. The first group of tests considered dependence on the *N* parameter. Number of premises were fixed in this case to $K=100$ and $minDY=10^{-2}$, which gives $C_{minDY} \approx 19$.

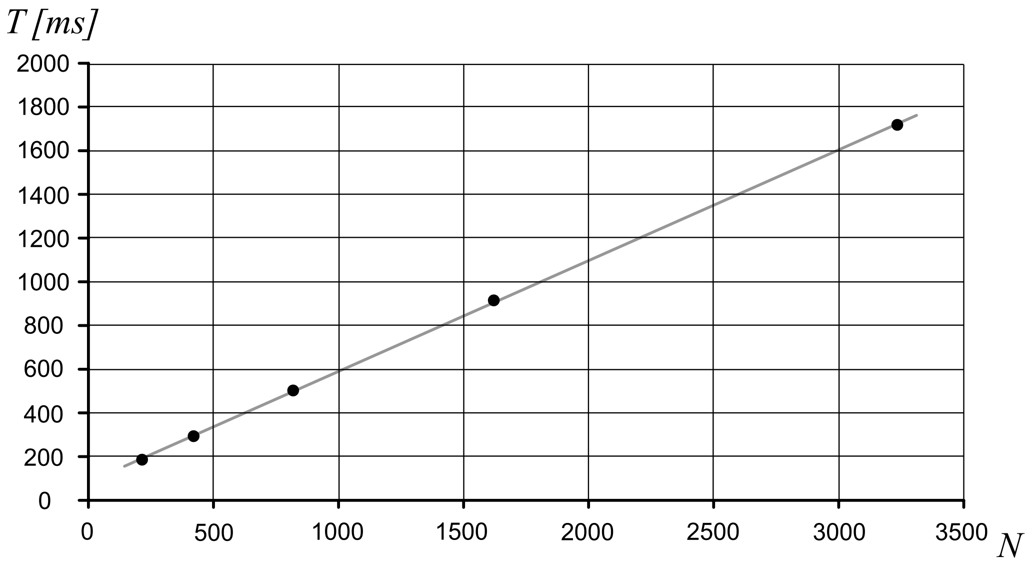

Fig. 5. Influence of *N* parameter (size of fuzzy sets description) on computation time for Baldwin's reasoning.

The second group of tests examined the influence of *K* on the time of computation. In this case *N* was fixed to 220 and *minDY* parameter stayed at the same level as in previous group of tests ($minDY=10^{-2}$).

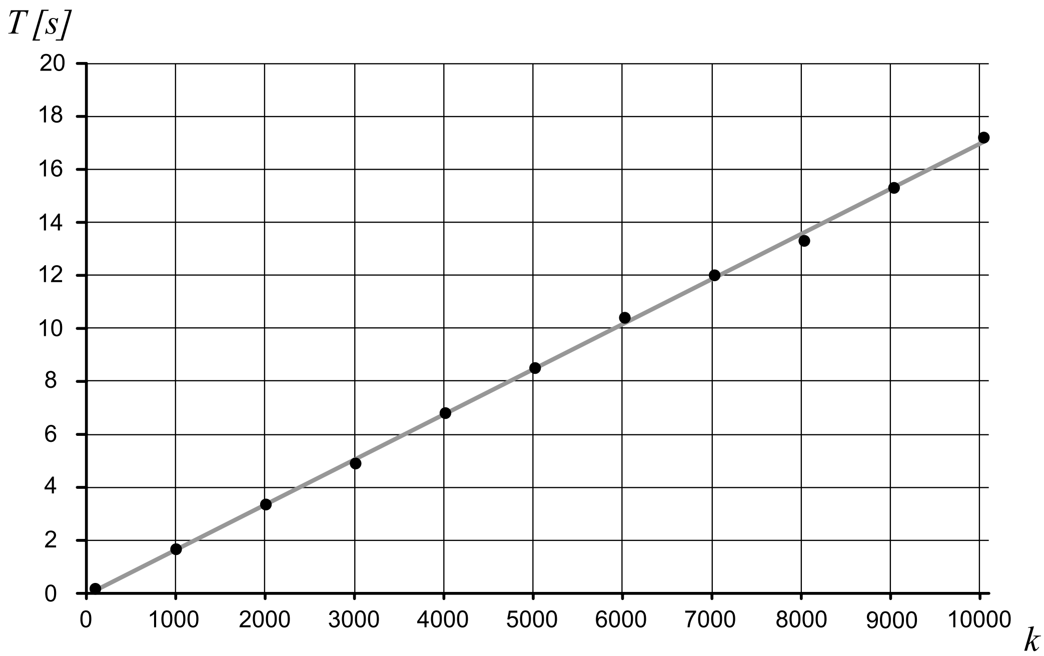

Fig. 6. Influence of *K* parameter (number of atomic premises in a rule) on computation time for Baldwin's reasoning.

The examinations directly show linear dependence according to *N* and *K* parameters, which confirm the assumptions leading to final complexity (14).

Although the approach of Baldwin is characterized by the same level of computational complexity as simple common solutions (considering dependence on *N* and *K* parameters) it is also dependent on C_{minDY}. This parameter is crucial to the final time of computation. The smaller C_{minDY} is, the less influence it has on (14) and the solution works faster. Additionally, (14) includes more sub-expressions then (9), which also can change the complexity in some cases.

3.3. COMPARING COMPLEXITY OF THE APPROACHES

Results presented on Fig. 4 and Fig. 6 let to compare the time complexity of the two analyzed methods. Computations for one rule containing 5000 atomic premises (*K*=5000) took approximately 16 [ms] for simple approach and 8,5 [s] for the implementation of Baldwin's method. Considering the parameter *N* fixed respectively at the level of 2023 and 220 and linear dependence in both cases, it lets to calculate a relative rate of performance *P*:

$$P = \frac{\left(\frac{8500}{220}\right)}{\left(\frac{16}{2023}\right)} \approx 4900, \tag{15}$$

which indicates, that the full implementation of Baldwin's solution is in this case almost 5000 times slower then the fastest, simplified approaches. However, it needs to be emphasized that the analyzed method based on a fuzzy truth value was implemented without any simplifications and includes fuzzyfication of the input data. The full approach of Zadeh, based on the compositional rule of inference (1),(2), is characterized by exponential complexity depending on a number of atomic premises (*K*). That is the main reason why it is not applied in this form.

Computational complexity of simplified approaches considering input fuzzyfication could be described by the extended version of (9):

$$K(N + 2N) + (K - 1) + N, \tag{16}$$

where the *K*(*N+2N*) part is responsible for fuzzyfication of *K* facts and finding the highest value of their intersection with corresponding *K* premises. Considering large number of *K* the first part is dominant and the modification significantly increases the complexity from $K(\log_2 N)$ to 3*KN*. Assuming *N* at the level of 2023 and *K*=5000 the performance rate *P* of the simplified solution with input fuzzyfication in comparison to the system without it can be assessed as follows:

$$P = \frac{3KN}{K(\log_2 N)} \approx 550, \tag{17}$$

which means that the approach becomes in this case approximately 500 slower. Therefore, analyzed implementation of Baldwin's approach becomes only, more or less, 10 times slower in computation for presented case. Such difference is much more promising for the future research on optimized or simplified solution of Baldwin.

3.4. CONCLUSION

Analysis presented in this paper shows that the two considered approaches to fuzzy inference are characterized by linear computational complexity according to the number of atomic premises as well as the size of membership functions' description.

Results of performed examinations revealed a colossal difference in computation time between the common approach and the implementation of Baldwin's solution. However, an analysis considering fuzzyfication of input data (fuzzy facts) showed that the method based on a fuzzy truth value has the potential of possible applications in the area of systems with large knowledge bases.

The analysis of computational complexity for Baldwin's reasoning indicated that obtaining subsequent truth functions have the greatest influence on computation time. Algorithms constructing truth function of a premise and compound truth function are the most important in case of fuzzy systems containing rules with many atomic premises. The most improvement could probably be obtained for simplified methods with constant and small number of elements describing truth functions. Therefore, the future research should focus on these areas.

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