MODELLING THE DATA RECORD OF A PATIENT WALK 
BY LANGRANGE-POLYNOMIAL METHOD

Various options available in PSW footprint and walking characteristics measuring equipment [6], [7], give the user many aims in putting diagnosis. A Conclusion-Making Unit (CMU) that has been described in this paper supports the diagnosis automation procedures. Due to simplifying the CMU training process some affords in a field of the input record length reduction have been undertaken. The paper describes an analytical method of the data record description that allows converting discrete data samples into continuous function. This way a redigitalisation of the record can be done, where sampling period is matched with the walk length. This normalisation allows reducing the data record length used for fast training of the CMU.

1. INTRODUCTION

Medical experts use various methods [6], [7] and tools [2] allowing analyzes all mobile mechanisms of a patient body. The computer diagnostic equipment [5], [3] with a built-in conclusion-making unit (CMU) is successfully used in many fields of medicine. One interesting problem that concerns the CMU training time reduction has been discussed in this paper.

The data records being the analysis subject are collected by interface of Parotec System for Windows (PSW), the equipment described already in several works [6], [7], [2]. The data record is read by set of sensors installed in a shoe-insole [2]. The data available in this record is shown in various interfaces providing the user with many components of diagnosis. One of the most valuable concerns the walking trajectory analysis. The measuring unit collects the source record of static and dynamic data (while standing and walking, respectively). The PSW device consists of:

- a single-chip microcomputer measuring unit, reading a pressure distribution among sensors installed in the insole, as it is shown in the example interface in Fig. 1,
- a PC software package used for reading the data from the controller and for the data record visualization.

The user can put his first assumptions to the diagnosis, being a rough estimation of the walk abnormality, namely a class of the disease. After this elementary recognition the next diagnosis procedure concerns dynamic part of the data record. An abnormality of a time characteristic distri-
bution during the walk cycle allows the PSW user finding reasons of the patient health troubles, underlying the disease nature.

Fig.1. The example pressure distribution on a foot in a dynamic part of data.

2. THE CONCLUSION MAKING UNIT

The data extracted from the record can provide the users with several additions that need the automatic conclusion-making unit.

The PSW interfaces allow:
- analysis of body balance and mobile mechanisms,
- time measures analysis in a cycle of walk or run,
- recognition then monitoring complex neurological factors of the disease.

Doubts concerning diagnosis are reasonable smaller when it is supported by knowledge of the disease, classifying the disease characteristic features, given by a current data record.

The Conclusion Making Unit (CMU) allows comparing the selected part of the record with pattern images by filtering and extracting formulas [6], [7]. Very troublesome problem concerns number and quality of records used for neural network training.

Our early experiments with neural networks proved that the computer interfaces like PSW make the medical experts many troubles in putting a proper diagnosis. What is more the CMU was unable to generalize the disease features recognition, in spite of very long lasting training process.

The PSW data record consists of discrete pressure samples recorded on every sensor. The pressure values are measured on 24 sensors with sampling intervals defined by micro-controller internal clock. Although operator, in range of 100 to 200 Hz can select the sampling frequency of the data recorder, a random walking speed of the patient produces different lengths of the data record.

The neural network training process efficiency depends on precise classification of the disease features, while the sampling frequency of the data recorder does not change the characteristic fea-
tures of the disease. Anyhow grows density of the data record samples and the data sequence length.

Various sampling frequencies of the sensor-signals reading are explainable when we want to distinguish different activities as running or walking. Not sufficient samples in data sequence for running means that a part of important data is excluded. Similarly when the sampling frequency for walking activity is to high a data record describes the disease with higher resolution then it is needed. Using to long data sequences for neural network training a time of this experiment grows without any reason.

Statistically variable speed of a patient walk, with a constant clock period of the data controller, does not allow unifying the measuring conditions for every disease and for every patient.

A goal of the presented works was normalization of every data record that is used for the CMU training.

3. THE POLYNOMIAL APPROACH TO THE PRESSURE SPECTRUM APPROXIMATION

Let us assume that the controller-clocking unit is sampling pressure values of first five steps, 50 times on every sensor of 24. It means that the data input vector contains:

\[ l_p = 24 \times 50 = 1200 \text{ components}, \]

Each of these components contains 50 samples creating well-ordered set of values:

\[ X = \{x_i : i = 1,2,...,50\} \]

where: \( x_1 < x_2 < \ldots < x_{50} \), are values of input variables controlling a set \( Y_j \) of output variables. The discrete values of pressure in each interval \( i \) for every sensor \( j \):

\[ Y_j = \{f_i : i = 1,2,...,50\} \quad j = 1,2,...,24 \]

Before the sampling period normalization of the input record will start, the discrete data set has to be converted into continuous expression. The transformation formulas are defined in a following manner:

\[ F_j : R_x \to R^+ \cup \{0\} \quad R_x = \{x : x \in R \land x_i \leq x \leq x_{50}\}; x_i, x_{50} \in X \]

\[ F_j(x_i) = f_i \quad x_i \in X; f_i \in Y_j; j = 1,2,...,24 \]

The discrete pressure approximation procedures are found in several classes of algorithms:

- polynomial interpolation methods,
- quotients of polynomials - rational functions,
- trigonometric interpolation,
- spline interpolation.
The paper presents a polynomial interpolation method by means of Lagrange functions. The approximation algorithm experiments concern four data record classes: physiology, bunion pathology, ischialgia, lateral sciatic neuralgia. For every data class the most representative data record (pattern) was selected.

The Lagrange interpolation \[8\] equation has been defined as:

\[
L_n(x) = \sum_{i=0}^{n} y_i p_i(x)
\]

where:
- \( n \) – the polynomial degree,
- \( y_i \) – the polynomial value in \( i \) – node,
- \( p_i(x) \) – is an equation:

\[
p_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_{n-1})(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_{n-1})(x_i-x_n)}
\]

where: \( x_i \) – concerns an \( i \) node.

The calculation complexity of this interpolation algorithm expresses the equation \( O((n+1)^2) \) [8], where \( n \) – is the polynomial degree. For 50 data nodes the interpolation calculations complexity is equal to \( O(2500) \) that is simple task for an average PC.

When grows the polynomial degree the approximation quality seems to be better. Our experiments proved that the degree of polynomial has to be selected in a special way.

The simulation experiments we started from polynomial’s degree \( n = 49 \). The polynomial values in sampling nodes were calculated properly. Anyhow between nodes \( (x \notin X) \) the calculations were very unstable, with unexplainable values of the data record (eg. values 1000 higher then maximal pressure on sensors). This non-stability is observed for polynomials of degree \( n \geq 16 \).
Bellow this level the approximation procedures offer more satisfying characteristics of the pressure distribution functions.

4. AN ERROR ANALYSIS OF THE INTERPOLATION METHOD

The effectiveness of the interpolation measures have been supported by an error analysis. In classical approach to error analysis, in polynomial approximation methods, we assume that the functions being a subject of the interpolation procedure are differentiable in multiplication error equal to \((n+1)\) [8], [9], [11].

As in the presented subject the above condition is not available this criterion can not be used. That is why the absolute and relative errors-analysis is used for values in sampling nodes only.

According to the literature [9], these fault formulas are defined as follows:

\[
E_i = |f_i - F(x_i)| \quad i = 1, 2, \ldots, 50
\]

(7)

\[
RE_i = \begin{cases} 
\frac{E_i}{f_i} \quad \text{when } f_i \neq 0 \\
E_i \quad \text{when } f_i = 0 
\end{cases} \quad i = 1, 2, \ldots, 50
\]

(8)

where: \(f_i\) – concerns values in points \(x_i \in X\),
\(F(x_i)\) – is the approximation result (for \(x_i \in X\)).

Faults for single sensors are defined by following formulas:

average absolute error:
\[
E_j = \frac{\sum_{i=1}^{50} E_i}{50}
\]

(9)

average relative error:
\[
RE_j = \frac{\sum_{i=1}^{50} RE_i}{50}
\]

(10)

maximal absolute error:
\[
E_{\text{max}, j} = \max \{E_i : i = 1, 2, \ldots, 50\}
\]

(11)

maximal relative error:
\[
RE_{\text{max}, j} = \max \{RE_i : i = 1, 2, \ldots, 50\}
\]

(12)

where: \(j = 1, 2, \ldots, 24\).
The error analysis technology was used as the approximation effectiveness measures concerning various degrees of polynomials. The best approximation results for values between nodes we obtained for polynomial degree equal to 10 (results presented in table 1).

When grows the polynomial degree the approximation procedures become unstable at the end of the interpolation zone (Fig. 3, Fig. 4, Fig. 5 and Fig. 6).

A mathematical literature calls this problem Phenomena of Runge (PR) [4]. Anyhow the interpolation results closer to the central point between the nodes distance are satisfying even for higher polynomial degrees.

Trying to eliminate the PR the traditional interpolation algorithm was used in three independent sub-intervals between nodes distances. This approach is called multi-range interpolation algorithm for sub-intervals [4].

The presented approach provides the user with simple approximation algorithms and satisfying results, where the calculation complexity grows not remarkable.
### Tab.1. Absolute error values for Lagrange polynomials of 10, 12 and 14 degrees

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<tr>
<th>Sensor number</th>
<th>aver. relative err.¹</th>
<th>aver. absolute err.²</th>
<th>max. relative err.³</th>
<th>max. absolute err.⁴</th>
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<td>s14</td>
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**sum** 28.9  51.6  96.9  11.2  13.1  19.1  904.8  1559.5  2934.0  94.5  140.8  282.3

**mean-value** 1.2  2.15  4.04  0.47  0.55  0.8  37.7  64.98  122.25  3.94  5.87  11.76

¹ average relative error
² average absolute error
³ maximal relative error
⁴ maximal absolute error
* polynomial degree
The pressure distribution at 4 sensors

Fig. 3. The full range of approximation for polynomial degree \( n = 10 \).

Fig. 4. The full range of approximation by polynomial of degree \( n = 12 \).

The pressure distribution at 4 sensors

Fig. 5. A zone interpolation with polynomial degree 3 (external sub-zones – bold)

Fig. 6. Approximation with polynomial degree \( n = 14 \) without sub-zones.
Two extreme zones with Runge Phenomena have been approximated by low degree polynomials (n = 3). The left sub-interval concerns samples from 1 to 10. The right sub-interval samples from 40 to 50. For the remaining (internal) part of the approximation range, belonging to an internal zone a higher degree of polynomials can be used.

For zone-interpolation approach, in range of external sub-zones, the minimal error of approximation has been noticed for the polynomial degree n = 3 (Fig.7). For internal part of the approximation zone the best approximation results have been obtained for n = 14.

5. CONCLUSIONS

When grows the neural network complexity and its training process is not controlled properly, the network collects many not important details that cause the record recognition difficult or not possible.

The experiments in clinics proved the CMU high ability of the disease recognition. The expert options combine the current data with groups of models the most relevant to the pathology.

The CMU effectiveness highly depends on a good organization of neural network training process. The algorithm shown in the paper presents possibility of the data-record reduction. This way reduction of time and complexity of the CMU training operations is possible.
BIBLIOGRAPHY


