geometrical modeling, 3D structures,
free-form surfaces, implicit surfaces,
representation of anatomical structures

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DEDICATED 3D GEOMETRICAL MODELS FOR REPRESENTING ANATOMICAL STRUCTURES

The representation of anatomical structures requires to take into account their topological features such as their morphological ones. Differential geometrical properties also have to be managed through the associated 3D models. Unfortunately, classical geometrical modeling approaches do not provide these possibilities. In this paper, we describe two original approaches that bring a solution to this problem for the representation of tree-like cavities – such as vascular ones – and complex organs – such as the heart. Medical applications were given as an illustration of the use of two new approaches discussed bellow.

1. INTRODUCTION

Free form surfaces (Bézier or B-Spline ones, for instance) are usually defined as sets of parametric patches computed by means of blending functions acting on a mesh of control points. “Control points” provide a powerful tool for managing local deformations, and “blending functions” guarantee their differential properties. But they do not provide any control on their global shape.

More recently, implicit surfaces based on skeletons and potential functions have been developed: they provide an efficient tool for controlling the topological and morphological features of geometrical models, but they do not permit any local control on their shape.

We have been developing research works in two directions:

– through the creation of a meta level for defining new parameters and new control points that are able to take into account morphological features when using parametric surfaces for the representation of tree-like cavities
– through the definition of a multi-layer 3D model that integrates morphological features by means of its inner layer and local deformation features by means of its outer layer.

3D geometrical modeling becomes an important stake for many medical applications such as diagnosis, surgical planning or medical driven interventions. These applications can widely take advantage of the use of 3D medical imaging, but displaying the relevant data distribution in 3D

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space is not sufficient. Geometrical features need to be extracted from these data and structured into geometrical models.

Classical geometrical primitives, such as planes, spheres and cylinders for instance, cannot express reliably the geometrical complexity of organs and anatomical structures. The problem of representing complex geometries is not specific to the medical field. Thus, many research works have been developed in this area, and especially on free-form surfaces representation, during the last forty years. Most of these works deal with parametric surfaces, but approaches based on implicit surfaces have also been emerging recently.

The geometrical models that have been proposed, for example in the frame of Computer Aided Design problems, only bring partial solutions to the organ representation problem and do not provide a global solution to the integration of all feature levels in a single model.

We first give a short description of these approaches, with a presentation of their advantages and their drawbacks. Then, we introduce two recent and original approaches that enable to take into account different levels of information in the model.

2. CLASSICAL APPROACHES FOR REPRESENTING FREE-FORM SURFACE

2.1. PARAMETRIC SURFACES

Parametric surfaces have interesting properties: they are easy to compute and they provide a coherent way for moving on them. This is especially interesting in the frame of Computer Aided Design and Manufacturing.

Most objects such as mechanical parts, ship hulls, airfoils or car bodies have a complex shape. Their representation requires the use of free-form surface mathematical models. Thus, because of the properties mentioned before, many research works have been developed in this field using the parametric formalism. In the sixties, it dealt to the formalization of Splines as interpolation surfaces on one hand, and to the definition of Bézier surfaces as interactive models driven by control points and blending functions on the other hand. In the early seventies, the definition of B-Splines provided a local control on the surface, the capability of having a very complex shape without increasing the computations and a link toward Splines. All these developments have been based on the use of polynomial or piecewise polynomial functions [1].

A few years later, a generalization of these models in the “Projective Space” has been proposed through the definition of Non Uniform Rational B-Splines (NURBS). This definition enables the representation of various kinds of surfaces in a single formalism. In the eighties, approaches based on the use of non-rectangular patches, such as Bézier-Gregory triangular ones, have been developed to enable the representation of more complex global shapes. But the set of constraints on corresponding control points usually leads to inextricable systems.

All these developments have been provided to solve problems related to surface regularity, smoothness and control of local variations. But none of them proposed solutions to more global features such as topological or morphological ones.

2.2. IMPLICIT SURFACES

Implicit models defined as equipotential surfaces have been widely developed since the early nineties. The main idea of these research works was to design geometrical models that enable the control of closed surfaces by means of their morphology. These models are defined as implicit surfaces characterized by a skeleton and potential functions [1].

A closed surface bounds a volume, and the role of the skeleton is to give a shape to this volume. The skeleton is considered as a source of potential and the implicit surface is defined as the set of points having a given potential.

The main interest of such implicit surfaces is that they give an global control on the topology and on the morphology of the shape. But they do not enable straightforward local deformations.

2.3. A NEED FOR UPPER LEVEL GEOMETRICAL MODELS

As it has been developed in the previous sections, research works have been focused in two main directions:

- One of them provides free-form surfaces based on a parametric representation that has very smart local capabilities but no global control.
- The other one provides closed surfaces with an interesting global control but without any local one.

None of these approaches is satisfactory for organs and anatomical structures representation because a control is required at every level:

- Some organs have cavities (topological features)
- They may have various shapes (morphological features)
- Their surface usually has local variations (geometrical features)

In addition, we may want to represent other entities that are not volumes as in the case of vascular lumen representation: it is a quite complex surface that has to be associated with a tree-like structure.

In the next sections, we present two original geometrical modeling approaches that enable the representation of:

- tree-like structures
- complex shape organs

3. A NEW GEOMETRICAL MODELING APPROACH FOR VASCULAR CAVITIES

Vascular cavities are mathematically associated with open surfaces whose boundary is made of a set of connected components, each of them being a closed curve (the section of an artery, for example). In addition, this surface has a global shape that can be described as a tree-like structure.

Many approaches have been proposed for representing them. One of them, for instance, has been described in [2]: the geometrical model is made of “generalized cylinders” (represented by B-Spline surfaces with circular constraints on their control points) and of “junctions” (represented by an assembly of Bézier-Gregory patches with constraints on their control points): this approach provides an interesting solution but the regularity of the surface is difficult to maintain, especially at the "junctions".

In order to avoid this problem, we have developed a new approach based on the characterization of a structural level on classical parametric surfaces and the invalidation of parametric domain areas to satisfy the topological constraints [3].

Let us consider a classical parametric surface, such as a NURBS one. The parametric domain is a rectangle bounded by the extreme values of the parameters (u and v). But this parameterization is not adapted to the description of the tree-like structure: this consideration led us to the need of defining an intermediate level – related to the shape structure – for the parameters as well as for the control points.

But first of all, we need to characterize the vascular structure topology. A vascular structure (e.g. the artery lumen) can be modeled as a surface whose edge is made of N connected components: for a single vessel N = 2; for a junction N = 3; etc. Thus, we need, as a first constraint, to use a surface model that has the same topological properties. We obtain these topological properties by invalidating (N-1) areas of the topological domain (the Nth connected component of the edge is the limit of the parametric domain).

An intermediate representation level establishes a link between the basic definition of the surface and the tree-like vascular structure:

- The "new parametric space" enables to move along and across sections of vascular structures; it is obtained through potential functions associated with the centers of the invalidation areas;
- The "new control points" are distributed according to the iso-parametric lines.

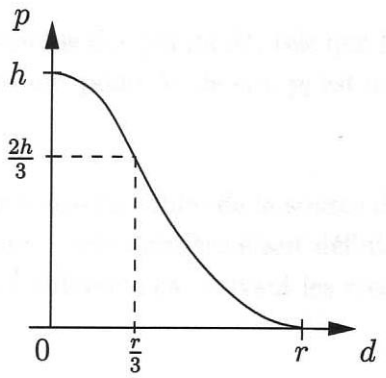
Let us start by considering a single vessel (N=2) or a single junction (N = 3). The method will be easily generalized to any tree like structure as we will see further.

Drawings below shows us the parametric domain (drawn in the (u,v) plane) and the new iso-parametric lines on which the "new control points" are distributed.

Let us go deeper into the description of the way we obtain this intermediate level.

For a single vessel (fig. 2.) and for a tree-like structure in general (fig. 3.), we would like to obtain the following parameterizations.

We obtain such a structure in the original parametric space by considering the potential lines generated by imbedded potential sources. In order to do that, we use finite support functions defined as follows:



$$\begin{cases} 0 \le d \le \frac{r}{3} & p_{(h,r)}(d) = 3h \times (\frac{1}{3} - (\frac{d}{r})^2) \\ \frac{r}{3} \le d \le r & p_{(h,r)}(d) = \frac{3}{2}h \times (1 - \frac{d}{r})^2 \\ d > r & p_{(h,r)}(d) = 0 \end{cases}$$

Then we distribute such potential sources along a line in the parametric space. We note that these sources may have different parameter values (h and r) as shown in figure 4.

The potential generated by all these sources, when they all have the same h and r parameters, but with various distances, can be visualized as follows (figure 5)

We note that the cols height (cols look like minima on the above profile) depends on the distance between consecutive sources. As a consequence, the global structure that characterizes the surface morphology can be represented by a set of distances between sources, as illustrated below (i.e. the C_1, C_2, … C_6 distribution along the x axis gives the morphology of the tree-like structure).

Finally, we can illustrate our approach by using a split view of the surface, each of its parts corresponding to an element of the structure and to an area of the parametric space (fig. 8.)

The pictures below illustrate the result of our geometrical modeling approach for a more complex vascular structure.

4. GEOMETRICAL MODELING OF COMPLEX ORGANS: THE HEART

Let us consider organs as made of biological tissues: they are volumes and thus, they can be represented by their boundary that is a closed surface. But this surface may be very complex as it is the case for the heart.

The heart has topological, morphological and geometrical specific features. Classical mathematical representations such as those described in section 2 are not sufficient to take into account all these features.

We have designed an original approach to bring a solution to such a problem [4]. This approach is based on the use of a three-layer geometrical model:

- The inner layer plays a role similar to the one of the implicit surface skeleton; it controls the object topology and describes its main morphological features.
- The outer layer plays a role similar to the one of a mesh of control points; it enables local variations on the surface
- The intermediate layer gives an articulation between the inner and the outer layers; it propagates the transformations of the inner layer to the outer one and not the inverse, except if these transformations bring important shape modifications.

The inner skeleton can be considered as the homotopic kernel of the shape. It is a 3D complex, i.e. a set of d-simplexes (d=0,1,2,3) that are points, segments, triangles and tetrahedrons.

The outer layer is used to locally modify the shape of the object. The drawings below illustrate the relations between these three layers on a 2D example.

In the frame of a reconstruction problem, we have data extracted from medical images and we want to build a geometrical model that can express the structure and the properties of these data.

Data are distributed on a regular grid. We project these data on a lower resolution grid, we compute the associated octree, and we use a thinning algorithm on this octree. Finally, we compute the complex that represents this eroded shape. The result is the inner layer.

We consider the inner layer as the skeleton of an implicit surface, and we compute the potential function that globally produces the best approximation of the set of boundary voxels (what is usually called the “crust” of the binary volume): it produces the transition layer.

The outer layer is computed from the transition layer to match as well as possible to this "crust". It is represented as local variations on the transition layer.

The images below show the result of this geometrical modeling approach for the representation of the heart.

5. CONCLUSION

In this paper, our purpose was to show that a purely mathematical surface definition is not sufficient to represent complex shapes as those of organs and anatomical structures. Thus, in the two new approaches described in it, we have emphasized on the fundamental elements introduced in the model that are the structure of the object and the different levels of geometrical information.

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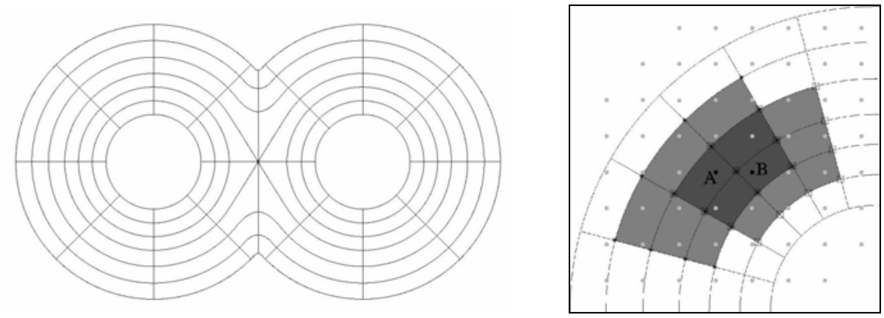

Fig.1. The intermediate parametric domain and set of control points

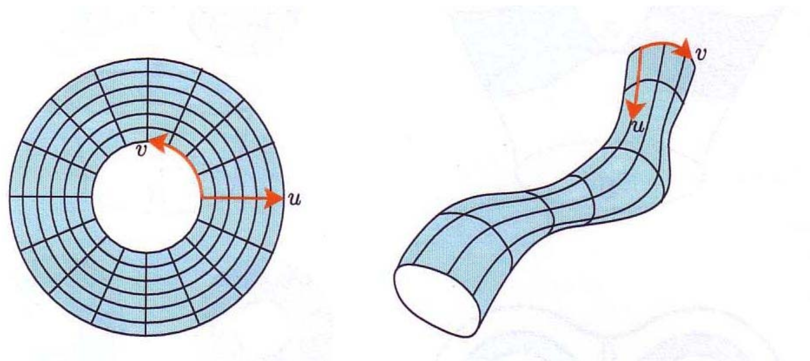

Fig.2. A new parameterization for a single vessel

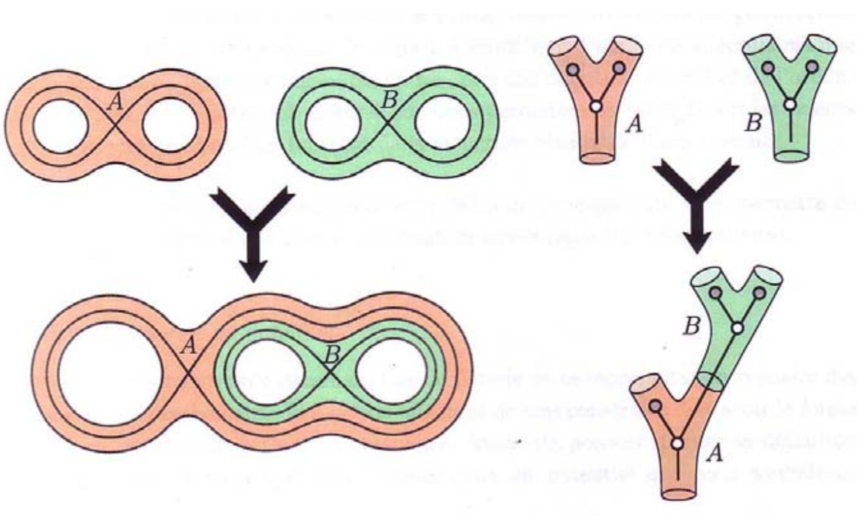

Fig.3. Generalization to a tree-like structure

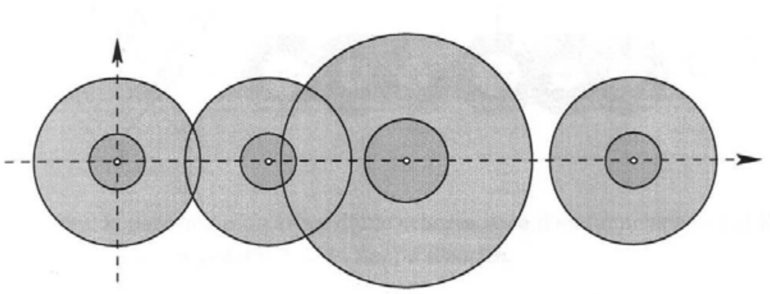

Fig.4. Distribution of potential sources

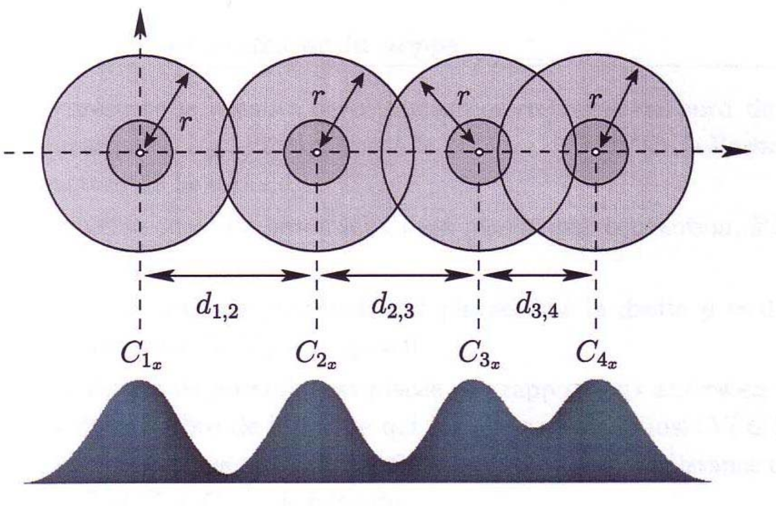

Fig.5. The global potential shape as a function of distances between potential sources

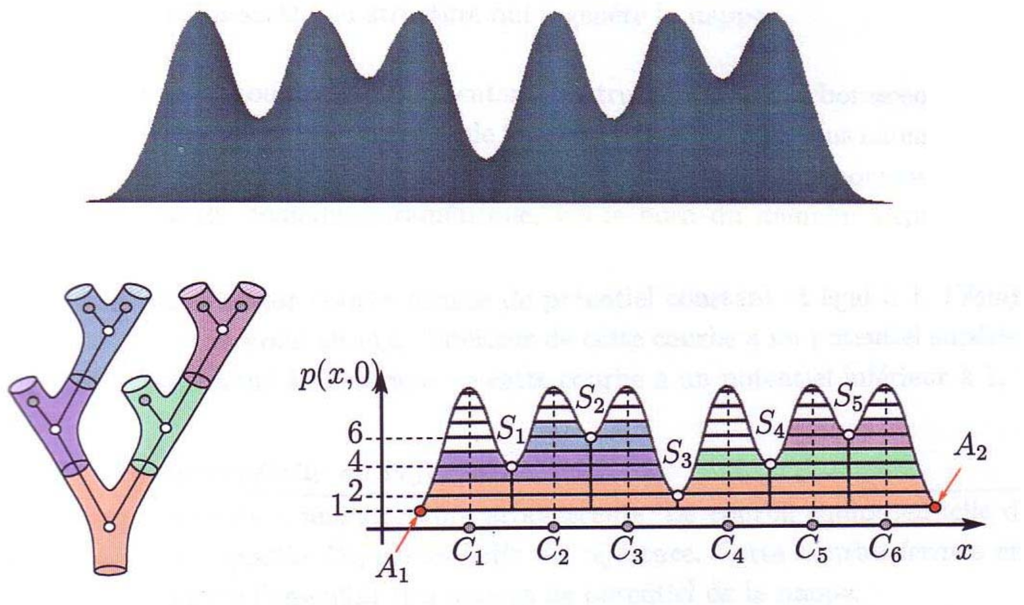

Fig.6. Potential generated by sources with the same parameters, colors being associated with the structure elements

The following picture gives us the labels associated with the (N-1) leaves of the tree.

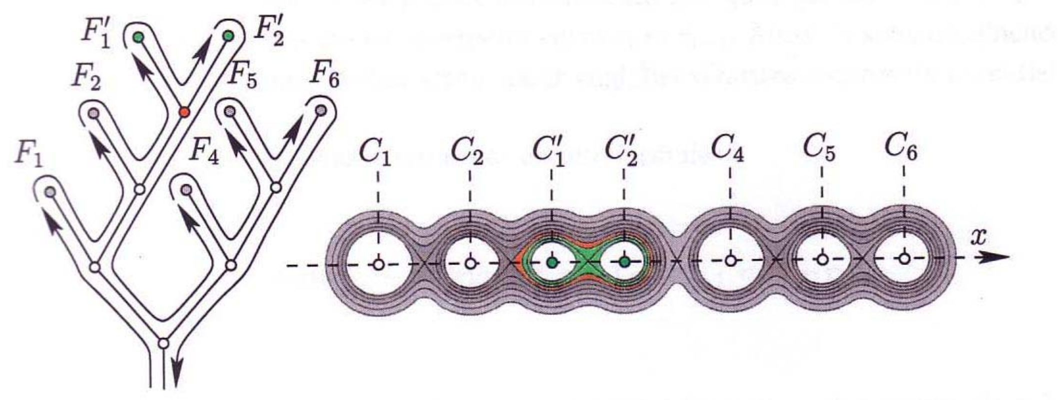

Fig.7. The complete description of the tree-like structure in the parametric space

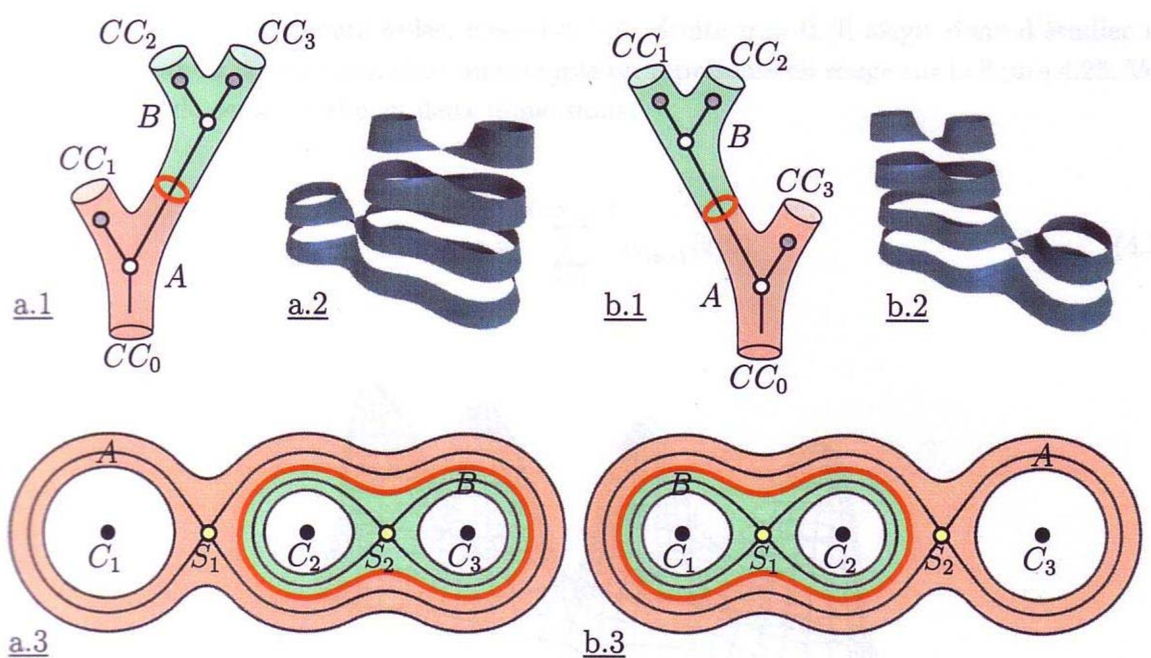

Fig.8. A split view of the generated surface (in blue)

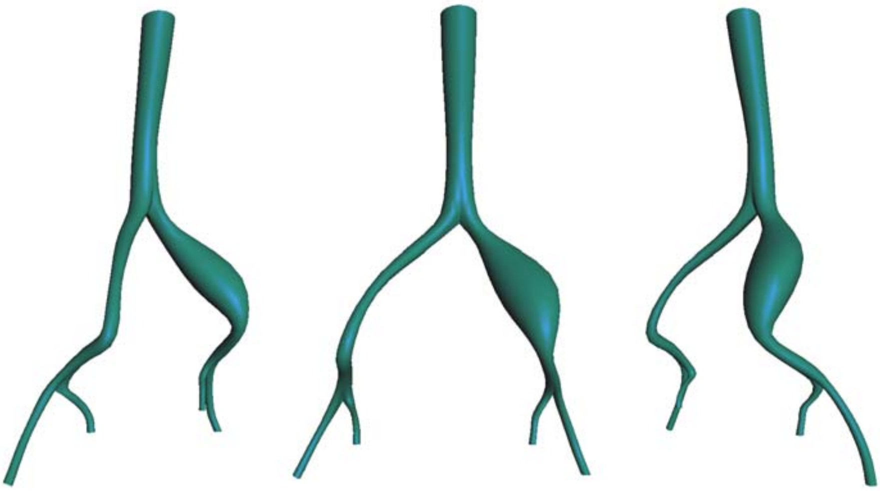

Fig.9. A complete vascular structure modeled by a single parametric patch

Fig.10. The elements of the inner skeleton

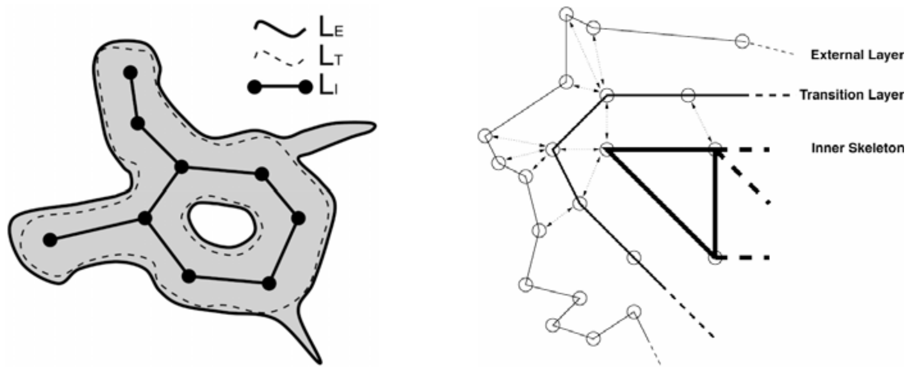

Fig.11. Relations between the three layers

Fig.12. The initial data, the inner layer, and all the layers with data