

*geometric modelling, multilayer shape representation,
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APPROACHES DEDICATED TO THE MODELLING OF COMPLEX SHAPES APPLICATION TO MEDICAL DATA

The way to model complex shapes has a significant influence depending on the context. Handling an object can be considerably increased if a good underlying model is used. On the contrary, preponderant problems can appear if an unsuited model is associated to the object. The main criterion to discriminate existing models is to determine the balance between: their ability to control global characteristics and the possibility to handle local features of the shape. The fact is very few models are adapted both to structure and to geometrical modelling. In this paper, we first describe an overview of existing approaches. They can be classified principally in two groups: skeleton based models, used to control the global aspect of the shape, and free form models, used to control local specificities of the object. Then, trying to keep the advantages of both techniques in mind, we present an original approach based on a multi-layer model to represent a 3D object. We focus on the ability to take into account both global and local characteristics of a complex shape, on topological and morphological levels, as well as on the geometric level. To do that, the proposed model is composed of three layers. We call the boundary mesh the *external layer*, including a multi-resolution feature. We enhance this representation by adding an internal structure: the *inner skeleton*, which is topologically equivalent to the input object. In addition to that, a third layer links the structural entity and the geometrical crust, to induce an intermediary level of representation. This approach is applied to classical and medical data through a specific algorithm.

1. INTRODUCTION

There are several means to represent a shape. Each of them has specific advantages and drawbacks. In particular, when the object presents a complex structure, the choice of the related model's formalization can focus either on the realistic visualization, on the structure, or on the physical behaviour. Our reflection and our work are based on this idea, and on the wish to elaborate a geometric modelling approach that could integrate the benefits of two main categories of formalisms. The aim being to easily characterize the model of a closed surface, we found interest in being able to extract the object's topological, morphological and geometrical properties independently.

Following a reflection on the two main classes of approaches: skeleton based models and free-form surfaces, which are manipulated with control points, we can point out the following remarks. On the one hand, the modelling approaches stand on the vocation to represent the shape globally, and on the other hand, on the vocation to define the object in a

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local way. The first class allows us to determine the structure of an object, limiting the surface considerations. The second class permits to control the boundary of the shape precisely, but often neglects the general vision we expect. Most of the modelling techniques compensate these lacks by adding features that make the underlying model more complex, but none of them really takes into account both the global and the local specificities of the shape.

In this paper, we focus on designing a modelling method that integrates the local geometric characterization of one class, and the ability to represent the topology and the morphology of the other class.

As an application, we use this specific model to reconstruct a set of points. On the opposite of classical methods, our aim is not only to characterize the boundary of the related object. Even if this is sufficient to represent the solid, we want a topological and a morphological descriptor of the object, as well as a coherent link between the various structures. To do that, we introduce three layers whose roles are to take into account these features.

In section 2, we develop some reflections on the properties that should be required for the geometric models. We skim over different models' formalizations, to extract the key characteristics of our approach. In section 3, we define the model by developing its three main entities: the *inner skeleton*, the *external layer* and the *transition layer*. Then we detail the reconstruction process related to the model. We finally validate the approach with examples (one of them being from real medical data) in section 4.

2. OVERVIEW OF EXISTING APPROACHES

In this section, we first compare two main approaches in geometric modelling: the surfaces defined by skeletons and the free-form surfaces defined by control points. Then we describe an overview of the similar techniques that contribute to the object's representation in the same way as defined previously, i.e. by considering global and local specificities of an object. Finally, we discuss the advantages and the drawbacks by pointing out a relevant compromise.

2.1. IMPLICIT SURFACES VS. CONTROL POINTS

2.1.1. IMPLICIT SURFACES AND SKELETONS

Most of the skeleton based techniques use the formalism of implicit surfaces. These surfaces, whose skeleton is usually a set of geometrical primitives, have been more and more employed in computer graphics over the past 15 years [5, 36, 6].

The implicit surfaces have several advantages, such as providing a compact and volume-based representation of the object. Moreover, the skeleton supplies the topological prior assumption and the structural information. This feature allows us to model complex shapes with arbitrary topology. The main asset of the implicit formalism lies in its ability to

control the shape *globally*, thanks to the skeleton that is centered in the shape. A simple modification of this entity is enough to induce a global deformation.

However, these approaches suffer from the following drawbacks: first it is uneasy to move on such a surface. The formalism only allows us to know whether a point is inside, outside or *on* the surface itself. Secondly, these surfaces are rather dedicated to represent smooth objects, sharp edges being more difficult to obtain. Finally, the characterization of details implies to take into account a large number of primitives. This major drawback considerably slows the model and becomes an obstacle for practical applications.

All these arguments point out the fact that it is difficult to get a *local control* with such surfaces because of the growing number of primitives to consider.

2.1.2. SURFACES DEFINED BY CONTROL POINTS

The most common formalism to represent a free-form object consists in using a parametric surface. Among these kinds of surfaces, defined by control points, there are the classical Bézier, B-splines and NURBS [2, 15, 27].

The advantages of this formalism are firstly due to the control points, which imply an intuitive and precise appreciation of the shape to model. Particularly, it is for this reason these surfaces are frequently used in CAD/CAM: they permit an intrinsic *local control* on the object's geometry. The second main advantage lies on the ability to move easily on the surface, thanks to the parameterization. Moreover, the very local notions of continuity, differentiation and curvature can be exploited in a coherent way.

However, surfaces defined by control points have main drawbacks. First of all, the modelling of objects with complex topologies (like with branching shapes or holes) keeps on being a traditional problem. Most of the time, we need to joint several patches. Depending on the case, this could be a serious problem. Indeed, these kinds of representations need to set a too important prior assumption in order to model general shapes. Thus, a collection of parametric methods can be found, dedicated to particular classes of objects (branching shapes [13], genus zero surfaces [32, 7]). In this context, self-intersection or continuity problems can occur if the control points are not located in a coherent way. This leads to the second major limitation: this kind of representation is hard to apprehend on a global level. Indeed, if we wish to deform an initial model by stretching it, we have to move the right set of points, and to verify the induced transformations on the shape. Although high-level operators exist to solve this problem (like *warping*), these manipulations are fussy and hard to calibrate (taking into account a small set of points or the whole object).

2.2. SIMILAR TECHNIQUES

We do not develop two alternative formalisms: mesh representations and superquadrics. The first one is powerful and widely used in computer graphics, it is the reason why we will employ it in our model (cf. section 3.1). The second formalism has the advantage to present a double parametric/implicit expression, but the local control is still not easy [35].

In this section, we will rather introduce approaches that emerge from the problematic to take into account both global and local characteristics of a 3D object. Although such approaches are not numerous, we can classify the existing ones in three categories.

2.2.1. REFINEMENT TECHNIQUES BASED ON CONTROL POINTS

In the aim to have a multilevel handling on free-form surfaces, [16] developed *hierarchical B-splines (H-splines)*. A subdivision process can be used to build local details on an initially rough surface. This refinement technique allows us to come across classical points addition, which increases considerably the number of control points. The H-spline solution also permits to have several levels of detail.

The *simplex meshes* are a good alternative to triangulation representations [11]. Such a mesh is said to be a k -simplex mesh if the points are $(k+1)$ -connected. Operators allow us to modify the topology and the geometry to increase the mesh resolution, and reconstruction algorithms can be adapted to this approach.

However, although these techniques are interesting on a geometrical level, they do not provide topological and morphological descriptors.

2.2.2. ENHANCED IMPLICIT METHODS

Among implicit techniques with skeletons, several take into account the local control on a 3D shape, via extended features.

The *skins* approach is located on the boundary of implicit techniques, multiresolution meshes and subdivision surfaces [25]. It is about a surface representation that lies on a particle-like model, used to sculpt free-form shapes interactively. These particles are constrained to maintain the mesh connected. This *skin* surrounds a set of skeletal primitives, to give a smooth surface through subdivisions. This approach can handle sharp edges. Local geometric details are added by subdividing the skeleton and by creating new primitives. However, performances are slowed down by the surface's definition by skeleton, and the topology can degenerate during the design of the shape, by doing self-intersections on the skin level. In spite of the multiresolution feature, this technique does not really permit to split the geometrical representation and the shape description (because the geometry is still defined by the skeleton, which grows when the level of detail increases).

To the same mind, the approach proposed by [21] is based on an implicit model with subdivision-curves skeleton. It is about an interconnected graph of curves, which generates an implicit surface by convolution (to avoid bulge problem). Increasing the level of detail amounts to subdivide the skeleton on particular points.

2.2.3. MULTIREOLUTION APPROACHES

Multiresolution techniques are frequently used according to the complexity of the geometrical details and the size of the data. Most of the time, it consists in representing the object within several levels of detail. The existing approaches can supply pyramidal structures [20], or multiresolution meshes, through vertex decimation [33, 34], vertex

clustering [31], edge contraction [29, 18, 28, 17], envelope simplification [10] or wavelets [14, 9].

Subdivision surfaces — In this frame, the formalism of *subdivision surfaces* is more and more used, as it presents a powerful multiresolution feature. It first appears following to the work of [8] and [12], and it is nowadays used into a large panel of applications in computer graphics [23, 37, 22, 19].

2.3. AN INTERESTING COMPROMISE

Further to this overview of the two main classes of approaches in geometric modelling, and of some existing techniques that characterize a shape through a collection of levels of detail, we put the following remarks forward:

- It should not be the role of the skeleton to take into account local variations on the surface, and yet its mode of computation and even its definition encourage it.
- It should not be the role of the control points to take into account global deformations depending on the structure. But usual modelling approaches force us to do so (through management attempts on an upper level of control point subsets).

The approaches with skeletons show that instead of considering the skeleton's instability, we should take into account that it is not well adapted to surface phenomena, but rather to shape description. The control points approaches provide a very good control on differential properties, but hardly any on topology and morphology.

We need a model that integrates in a coherent way the global and local characteristics of these two approaches.

3. A MODEL INTEGRATING GLOBAL AND LOCAL CHARACTERISTICS

3.1. THE MULTILAYER MODEL

The model must allow us to control these three concepts in a coherent framework: the topology (to be able to model complex shapes with no prior assumption), the morphology (including a shape descriptor) and the geometry (integrating a crust entity for precise handling). Moreover, according to the size of the data to model, we wish to include the multiresolution feature. We can define the aims of our modelling approach in two points: we want both the global structure and the boundary surface; and we want to detach the surface representation from the global shape's.

The model is composed of three layers (see figure 1-a). The first layer, the internal one, that we call *inner skeleton* (layer L_I). It defines the global structure of the shape, on topological and morphological levels. The *external layer* L_E characterizes the local variations of the shape's surface, regardless of the skeleton. The *transition level* L_T represents the articulation between the internal and external layers.

The goal of this split between local and global characterization is that the local perturbations on the surface do not deteriorate the global shape descriptor which is the inner

skeleton, and that the transformations on this inner structure are propagated on the external layer.

3.1.1. INNER SKELETON

The inner skeleton L_I is a *homotopic kernel* enhanced by morphological features. We define the inner skeleton L_I on a structural level as a *3-complex*, i.e. a set of tetrahedrons, triangles, segments and points. The edges define the connectedness relations between vertices. When three neighbours are connected, we obtain a filled triangle, and when four neighbours are connected, we obtain a solid tetrahedron (figure 1-b). A cycle of more than three edges defines a one-holed surface.

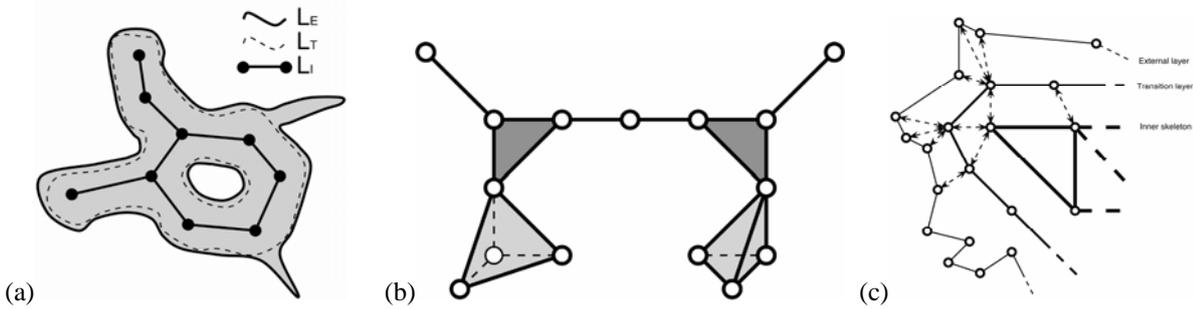


Fig. 1. (a) Scheme of the 3-layer model.
 (b) Example of 3D structure for the inner skeleton.
 (c) The three layers and the links between the primitives.

3.1.2. EXTERNAL LAYER

The external layer L_E is a simple triangulation. The vocation of this layer is to define the geometry of the object, given as a set of unstructured points. The multi-resolution feature is supported by this layer. Considering $L_E^{100\%}$ the maximal level of detail for this layer, the various resolution levels are defined by $L_E^{r\%}$, with $r_{\min} \leq r \leq 100$, r_{\min} being the percentage below which the layer becomes non-manifold.

3.1.3. TRANSITION LAYER

The transition layer L_T represents an intermediate geometrical level and a structural entity that makes the link between the global definition and the local characterization of an object. The inner and external representation levels are both as important and we want to characterize the articulation between them.

In the geometrical frame, we define the transition layer as an intermediary triangulation between the two other entities. It induces a structural link allowing us to go from one layer to another (i.e. an element of the external layer can refer to an element of the inner layer and vice versa). Furthermore, we set the equality $L_T = L_E^{r_{\min}\%}$ between the transition layer and the minimal resolution level of the external layer. The fact that L_T is the

most simplified level of L_E provides a natural evolution from $L_E^{100\%}$ to the transition layer L_T by mesh reduction in the reconstruction process.

Transition Graph — In addition to the previous geometrical definition, the transition layer includes a particular data structure: the underlying graph G_T linking the two skeletons allows us to set coherent relations within the object (see figure 1-c). The edges of this graph are defined by a *shortest distance* criterion [24].

3.2. RECONSTRUCTION PROCESS

In this section, we develop the reconstruction process related to the proposed model in order to express how the three layers are obtained, starting from a 3D set of points (general case of reconstruction). Using a digital volume allows us to skip a step, as it is a more particular case.

The main idea is to get an expression of the structure of an unorganized cloud of points, given as input data. We do not simply want to characterize the boundary of the shape with the external layer. Even if it is sufficient to represent the related solid, we attempt to exhibit a topological and morphological descriptor of the object. This point is fundamental, because efficient techniques of reconstruction are numerous, but they principally focus on the surface reconstruction without taking the structure into account.

The process is composed of two stages, themselves being most of the time well known techniques in computer graphics. However, concerning the first stage, we developed an original method to obtain the inner skeleton by doing a *homotopic peeling* applied to an octree.

The algorithm consists of two independent steps: the extraction of the inner skeleton and the characterization of the crust (from the external layer to the transition layer). We illustrate the process by considering the *Stanford bunny*¹ data in the initial form of a cloud of points (cf. figure 2-a). It is a simple object, of genus zero, but more complex examples are given in the next section.

3.2.1. SKELETON EXTRACTION

The inner skeleton extraction goes through a conversion of the data into the discrete space \mathbf{Z}^3 . We wish to characterize the topology and the global shape, by keeping only a small set of relevant voxels (or groups of voxels).

Step 1: Embedding the Cloud of Points into a Digital Volume

Let C be the initial cloud of points. We adopt the same principle found in [4] to embed C into a digital volume V (figure 2-b).

Step 2: Octree Conversion

The digital volume is then converted into an octree [26], to keep in mind the notion of *voxels grouping*. A single voxel does not represent an important morphological detail, but on the contrary, a block of voxels defines a large area that has to be included into the structure of the inner skeleton (figure 2-c).

¹ Available at the URL: <http://www-graphics.stanford.edu/data/>

Step 3: Interactive Thinning of the Octree

We adopt a classical thinning process, whose asset is to supply a homeomorphic entity related to the object. There is no algorithm dealing with octrees in such a way, so we use a simple algorithm initially designed for digital volumes, and we adapt it to work on octrees (by modifying the neighbourhood relationship and the local thinning criterion). The thinning (or the *peeling*) problematic first appeared in \mathbf{Z}^2 [30]. The principle is to delete the pixels that do not affect to topology of the object. Such points are said *simple*: when erased, no holes are created and no components are disconnected. [3] Extended this concept to \mathbf{Z}^3 . To extend this criterion to an octree, we defined in [24] the ω -neighbourhood as the equivalent to the 26-neighborhood in \mathbf{Z}^3 , and the $\bar{\omega}$ -neighbourhood as the equivalent to the 6-neighborhood (for the complementary object, i.e. the background). The algorithm we use to peel an octree is derived from the initial thinning process into a digital volume. In our case, the size of the octree elements intervenes.

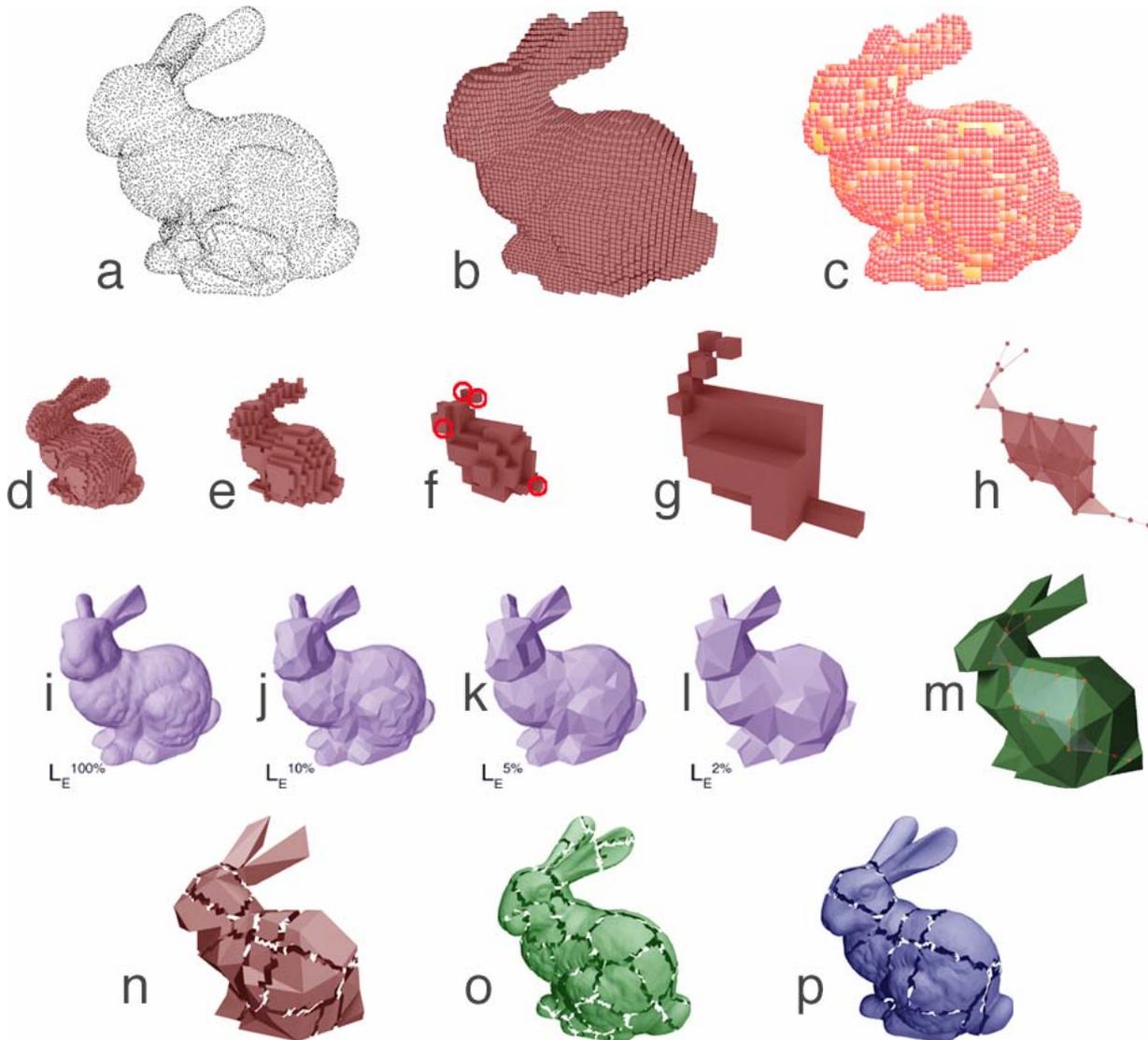


Fig. 2. (a) Initial cloud of points. (b) Digital volume. (c) Octree of the bunny. (d,e) Two peeling steps. (f,g) Setting four ending points. (h) Inner skeleton. (i,j,k,l) Some LoDs of the bunny. (m) The transition layer of the bunny. (n,o,p) Distribution of points among three layers.

The figures 2-d,e illustrate two steps of the bunny peeling with this algorithm. We enhance this algorithm by adding the interactive feature that allows the user to guide the process. It consists in setting representative elements, which contribute to the morphology as *ending points*. When the current element is such a point, it cannot be removed. The figures 2-f,g show the result when four ending points are selected by the user: the ears, the nose and the tail. The resulting octree is really homeomorphic to the initial object, and it supplies a good morphological characterization.

Step 4: Computing the Complex Related to the Reduced Octree

The last step to get the inner skeleton L_I consists in computing the reduced octree into a complex. This is done thanks to the ω -neighbourhood. Edges, triangles and tetrahedrons are created according to the adjacency of the octree elements (figure 2-h).

3.2.2. FROM THE EXTERNAL LAYER TO THE TRANSITION LAYER

Step 1: Polygonizing the Cloud of Points

The finest geometrical characterization of the external layer $L_E^{100\%}$ is a classical triangulation of the data points (see figure 2-i). We used the *Cocone* module which preserves the topology (described in [1]).

Step 2: Multiresolution and Transition Layer

The figures 2-j,k,l show some reduced meshes of the bunny, until $L_E^{1\%}$ which is the last step before the triangulation becomes non-manifold (cf. figure 2-m). We go from the external to the transition layer, and we set $L_T = L_E^{1\%}$ to define the geometrical characterization of the transition layer. All the mesh simplifications in this paper have been done using *QSlim* module [17].

Step 3: Computing the Transition Graph

To make the link between structural and geometrical levels of the model, the last step of the whole process is the computation of the transition graph G_T . The figures 2-n,o,p show the distribution of the points of L_T according to the points of L_I (n), the points of L_E according to the points of L_T (o), and the points of L_E according to the points of L_I (p).

4. APPLICATION AND EXAMPLES

In addition to the bunny's example, we go further into the validation of our approach by taking three other examples. The horse and the dragon (a one holed object) are classical clouds of points from the Stanford database, and the last example comes from medical imaging (a *fetus's heart*). Such an organ presents a complex structure. The table below illustrates the number of vertices according to the layers for each example. The figure 3 shows the inner skeleton, the transition layer and the external layer of the two first objects. The figure 4 illustrates each step of the whole process with the heart's data.

Number of vertices	L_I	L_T	$L_E^{100\%}$
<i>Bunny</i>	24	76	7030
<i>Horse</i>	97	244	48485
<i>Dragon</i>	63	108	44315
<i>Heart</i>	1310	1596	35260

5. FUTURE WORK

The good morphological properties of the inner skeleton could be used in a *shapes' recognition* module. The aim being to classify an object on topological and morphological criterions, the process could lie on a catalogue of typical objects arranged according to shape indications. This analysis perspective could be applied to computer vision, robotics, etc. The major work to do in the future is the *animation* of a reconstructed object. This can only be done if the object is well positioned (as an evidence, it cannot work on the bunny, as legs are not defined by the inner skeleton). For example, to animate a character expressed by the 3-layer model in a standing position (limbs being well defined), specific rules have to be determined to move external and transition layers after handling the skeleton.

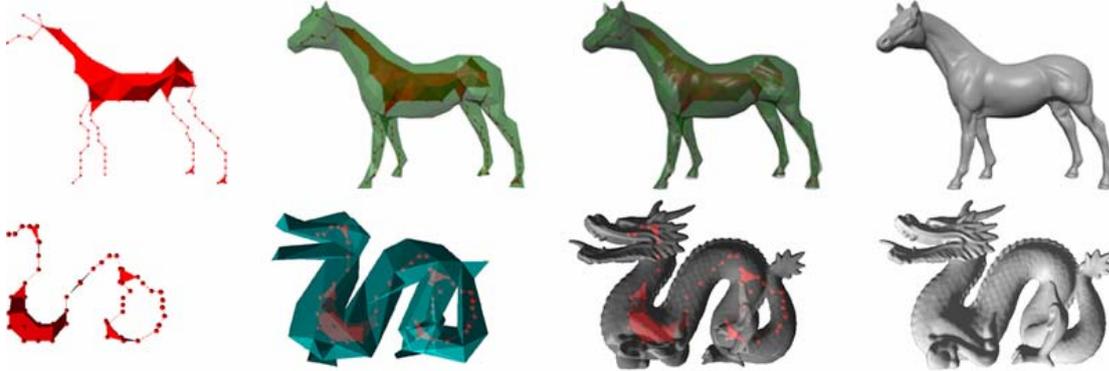


Fig. 3. Expression of the structure of the horse and the dragon within the reconstruction process.

6. CONCLUSION

We have presented a new shape formalism, which aims to give an explicit expression of an unstructured cloud of points, through three entities. The external layer defines the crust of the object in a classical way. In addition to that, the transition layer and, above all, the inner skeleton, allow us to get the structure of the object. This is done not only by characterizing the global shape, but also by a specifying a relevant topological entity. Moreover, we have validated our approach on typical data and on complex medical data.

We believe the underlying model can be applied to several applicative domains, taking part of the inner skeleton's assets.

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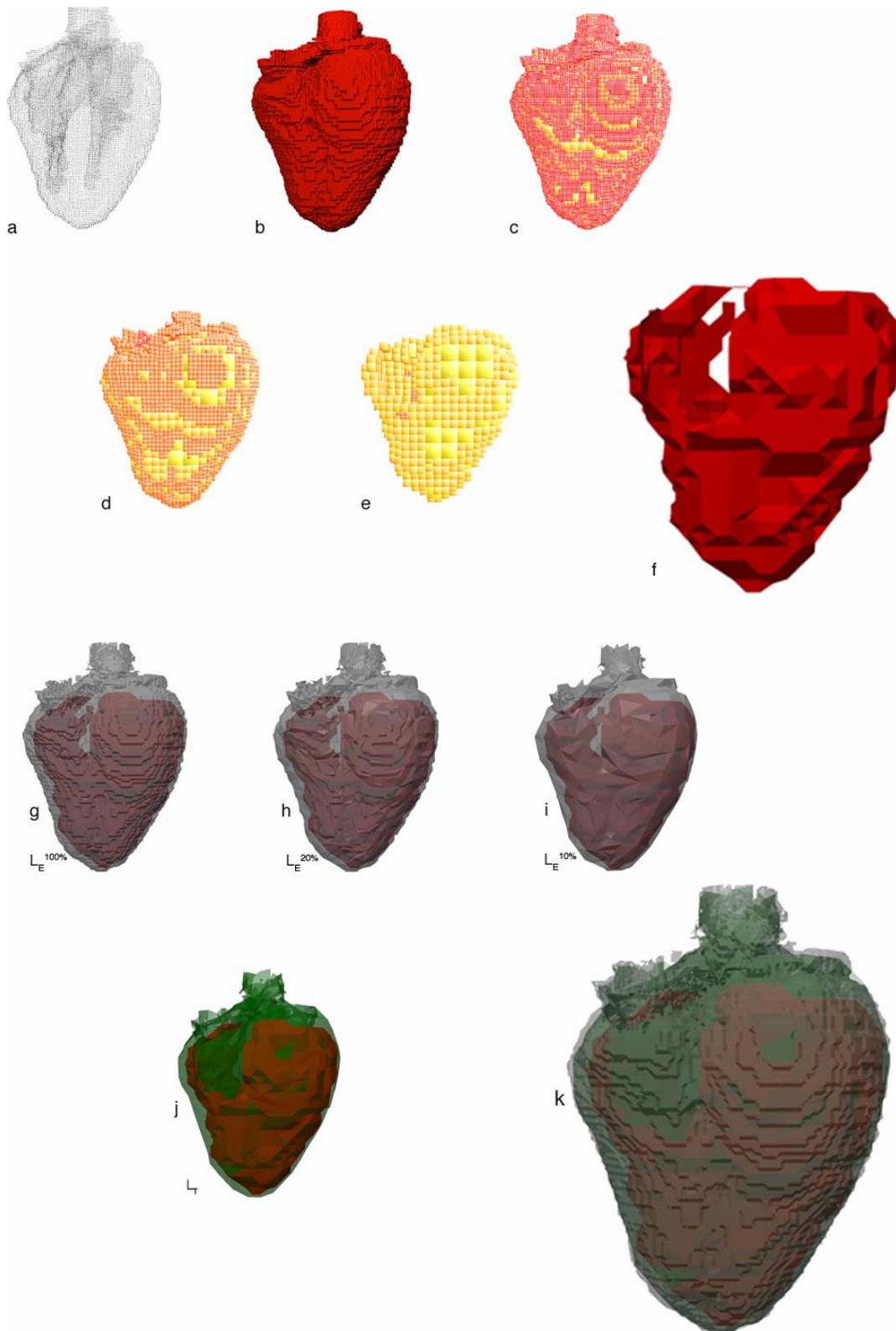


Fig. 4. Multilayer model of the fetus's heart.
 (a) Starting cloud of points. (b) Related digital volume. (c) Octree encoding. (d,e) Octree thinning.
 (f) Inner skeleton. (g) External layer. (h,i) Multiresolution. (j) Transition layer. (k) Final model.

